A MAXWELL THEORY FOR THE SCATTERING OF TM POLARIZED ELECTROMAGNETIC BEAMS BY A DOUBLE METALLIC SLIT

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Resumen.

Se presenta una teoría vectorial de la difracción de haces Hermite–Gauss sobre una pantalla metálica de espesor h con dos rendijas de ancho ℓ y separación d. los haces inciden oblicuamente con polarización TM. Estudiamos numéricamente los patrones de difracción en campo lejano como función de los parámetros opto–geométricos: longitud de onda λ, ancho de rendija ℓ, separación d, ángulo de incidencia θ_i y orden del polinomio de Hermite m. Por otra parte, en la región vectorial de la difracción dada por $\lambda/\ell > 0.2$ donde λ es la longitud de onda incidente, se analiza numéricamente la energía difractada a lo largo de la dirección del haz incidente (E_i) y la validez de la propiedad de la difracción escalar $E_i = N\tau/\lambda$ donde τ es el coeficiente de transmisión y N el número de rendijas.

Abstract

We present a rigorous theory for oblique incident Hermite-Gaussian beams, diffracted by two slits of width ℓ and separation d, in a thick metallic screen for the case of polarization TM(S). The far field spectra as a function of several opto-geometrical parameters, wavelength λ, slit width ℓ, separation d, incidence angle θ_i and Hermite order m is analyzed. In the vectorial diffraction region given when λ/ℓ >0.2, where ℓ is the incident wavelength and as a function of the separation between slits d; we have numerically analyzed: the far field spectra, the energy diffracted along the incident beam direction (E_i), and the validity of an approximate diffraction (scalar) property, namely $E_i = N\tau/\lambda$.

Palabras clave:

Difracción, Esparcimiento, Doble rendija.

Keywords:

Diffraction, Scattering, Double slit.

Introduction.

Currently there are several rigorous theories of diffraction by plane electromagnetic waves (Enriquez *et al.*, 2011) and Gaussian beams (Mata *et al*, 1993); (Mata *et al*, 1994) by two slits in metallic screens of zero thickness. However these theories do not treat with Hermite-Gauss or oblique incidence, nor thick screens of nonzero thickness (Mata *et al*, 2008).

In this paper, we present a novel rigorous theory of diffraction that allows to consider the illumination by Hermite-Gaussian beams at oblique incidence on two slits of width ℓ and separation d in screens with infinite conductivity and thickness h.

We analyze the coupling between slits through the numerical study of the diffracted energy along the direction of the incident (E_i) beam energy as a function of the parameter of separation d between the slits. It is revealed the existence of oscillations in the energy E_i . We also show that in the case of TM(S) polarization, the energy E_i is special because when compared to other diffraction patterns. Finally, we show that the scalar property valid at the scalar region (λ/ℓ <0.2) $E_i = N\tau/\lambda$ (Alvarez-Cabanillas, 1995) is no longer valid.

A Vector Theory of Diffraction.

In Fig.1 we have two slits on a screen of infinite conductivity, and non-zero thickness denoted by h. In this screen, you have two parallel to the Oz axis, ℓ wide and spaced slits d. The display is in the gap and impinges perpendicularly on it a Hermite-Gaussian beam with wavelength $\lambda = 2\pi/k$ and order m. We will use the complex representation for the fields and omit the time factor going forward $e^{-i\omega t}$. H is the magnetic field when you have the TM (magnetic field parallel to the axis Oz) polarization. The H field satisfies the Helmholtz equation (Mata *et al*, 1994).

$$\partial^2 \mathrm{H}/ \partial \mathrm{x}^2 + \partial^2 \mathrm{H}/ \partial \mathrm{y}^2 + \mathrm{k}^2 \mathrm{H} = 0. \qquad (1)$$

Denote by H_I the solution of Eq (1) in the region I ($y > h/2$), expressed by a plane wave expansion:

$$H_I(x,y) = \frac{1}{\sqrt{2\pi}} \int_{-k}^{k} A(\alpha)\, e^{i(\alpha x - \beta y)} d\alpha + \frac{1}{\sqrt{2\pi}} \int_{-k}^{k} B(\alpha)\, e^{i(\alpha x + \beta y)} d\alpha \qquad (2)$$

The first integral is identified with the incident beam due to the sign of the α and β k-components.

In region II, within the screen, $-h/2 < y < h/2$ the electromagnetic field will be represented by the following modal series:

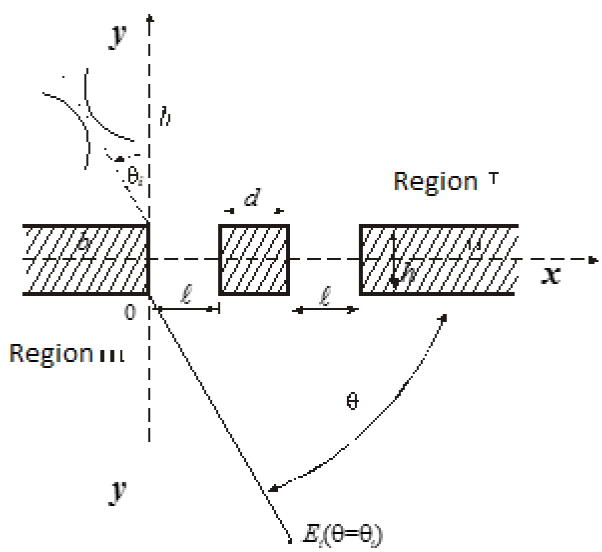

Figure 1. Our system. Two slits of width ℓ and spacing d in an infinitely thick conducting screen h. The energy diffracted along the incident direction (E_i) is diffracted in the direction of θ (relative to the axis Oy) = θ_i (From the axis Ox).

$$H_{II}(x,y) = \sum_{n=0}^{\infty} a_n^1 \varphi_n^1(x,y) + \sum_{n=0}^{\infty} a_n^2\, \varphi_n^2(x,y) \qquad (3)$$

Where in $i = 1{,}2$ the set $\varphi_n^i(x)$, are functions whose normal derivative is zero at the walls for the TM polarization.

The diffracted field below the screen, for $y < -h/2$ (region III), could be expressed as a plane wave expansion too:

$$H_{III}(x,y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}C(\alpha)\,e^{i(\alpha x+\beta y)}d\alpha \qquad (4)$$

Our goal is to determine the transmitted field (Eq. (4)), for which one needs to determine $C\,(\alpha)$. Note that $C\,(\alpha)$ depends on the coefficients a_n^1 and a_n^2 and the incident amplitude $A(\alpha)$. For this, we use the appropriate conditions of continuity, which could be obtained from Maxwell's equations (Alvarez-Cabanillas, 1995). These conditions lead us to the following matrix system, in which the matrix columns $\boldsymbol{a_1}$ and $\boldsymbol{a_2}$ are formed respectively by the coefficient a_n^1 and a_n^2.

$$\boldsymbol{M_{11}a_1}+\boldsymbol{M_{12}a_2}=\boldsymbol{S_1},$$

$$\boldsymbol{M_{21}a_1}+\boldsymbol{M_{22}a_2}=\boldsymbol{S_2}, \qquad (5)$$

where M_{ik} $(i,k\ =\ 1{,}2)$ are square matrices dependent on the opto-geometrical parameters and S_i $(i\ =\ 1{,}2)$ are matrices depending only on $A\,(\alpha)$. The determination of the modal coefficients a_n^1 and a_n^2. allow us to calculate the diffracted field in any region for TM polarization.

Results and Discussion.

Using the complex Poynting vector is possible to obtain the diffracted intensity at the angle θ. For a Hermite-Gaussian beam, the spectral amplitude is (Mata *et al*, 2008):

$$\mathrm{A}(\alpha)=\frac{\mathrm{L}}{2}\mathrm{i}^{\mathrm{m}}\mathrm{H}_{\mathrm{m}}\left[-\frac{\mathrm{L}}{2}(\alpha\sin\theta_{\mathrm{i}}-\beta\cos\theta_{\mathrm{i}})\right]\times\left[\sin\theta_{\mathrm{i}}+\left(\frac{\alpha}{\beta}\right)\cos\theta_{\mathrm{i}}\right]\mathrm{e}^{(-\mathrm{i}\alpha\mathrm{b})}\times\mathrm{e}^{[-(\alpha\sin\theta_{\mathrm{i}}-\beta\cos\theta_{\mathrm{i}})^2\mathrm{L}^2/8]} \qquad (6)$$

where H_m is the Hermite polynomial of order m. The position of the beam waist is given by the parameter b (see Fig. 1).

In the figures relating to energy diffracted along the direction of the incident beam is $E_i(\theta\ =\ \theta_i)$ the diffracted angle in the direction of the incident beam, measured from the axis Ox and θ_i is the angle of incident beams to the axis Oy measured. The energy, the diffracted intensity $I(\theta)$ and the transmission coefficient τ are normalized to the total incident energy I_0. All parameters normalized opto-geometrical width lof the slots ℓ, that is, ℓ= 1.

In Figs. 2 and 3 show the diffraction patterns of Hermite-Gaussian beams for the fundamental mode $m = 0$ at normal incidence and oblique incidence of $30\,°$; the wavelength of the incident beams is λ/ℓ=0.9, with extremely wide Gaussian beams L/ℓ= $500/\sqrt{2}$ and fixed at the position b/ℓ=0.5, the thickness of the screen is h/ℓ =1and the gaps between slits are d/ℓ= 0, 1, 3.5 and 5.

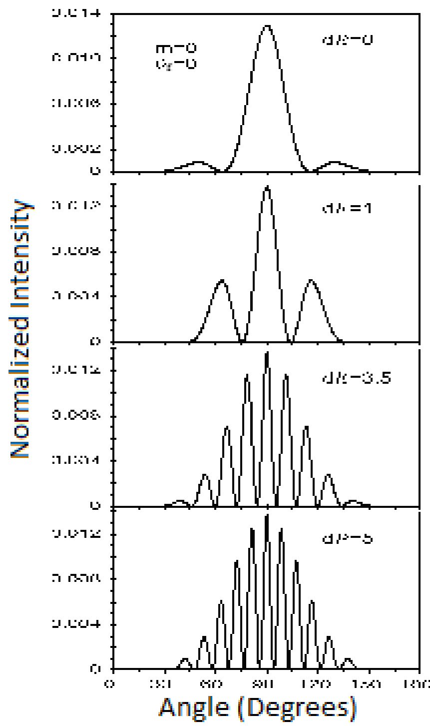

Figure 2. Diffraction patterns normalized $(I(\theta)/I_0)$ of Hermite-Gaussian beams of $m = 0$ normally incident on two slits so. With λ/ℓ = 0.9, L/ℓ=500/$\sqrt{2}$, h/ℓ =1 and position b/ℓ=0.5 and for separations d/ℓ = 0, 1, 3.5 and 5.

The shape of the diffraction patterns for the $m = 2$ mode, not shown, is identical to the spectra of FIGS. 2 and 3 (with the same opto-geometrical parameters) except for a scaling factor which provides a lower intensity for this mode, from the respective Hermite polynomial.

From these diffraction patterns, we have taken the diffracted energy E_i along the direction of the incident beams. Figs. 4 and 5 show the behavior of the E_i separation according to d for $m = 0$ and 2 modes; opto-geometrical parameters of these figures are the same in Fig. 2 and 3.

The curves of FIGS. 4 and 5 show the oscillatory behaviors as E_i a function of the spacing d, for the period is normal incidence to oblique incidence λ and the period is 2λ.

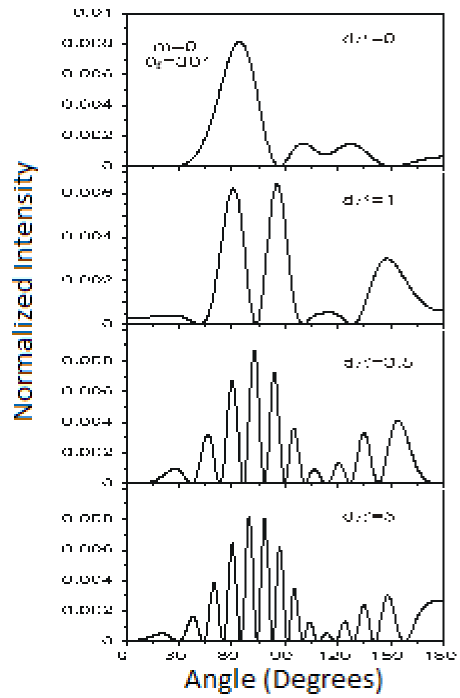

Figure 3. Standard diffraction patterns $(I(\theta)/I_0)$ Hermite-Gaussian beam for $m = 0$ to 30 ° obliquely incident on two slits so. Same parameters of Fig.2.

In Fig. 5 has also been drawn in broken lines the 2τ/λ function. As you can see, this function does not overlap with the energy with E_i which we can say that the property of diffraction E_i =2τ/λ is not valid in the vector region at least for the separation parameter d and doing extremely wide.

Finally, in Fig. 6 different diffracted energy around the energy is E_i. The upper curves of Figure 6 correspond to normal incidence for the $m = 2$ mode, with the same parameters of Fig. 3; diffracted energies correspond to the angles diffracted $\theta = 90°, 91°, 92°$ and $94°$. The curves in the lower window of Fig. 6 correspond to oblique incidence of 30°, also for mode $m = 2$, with the same parameters of Fig. 4. The diffracted energies shown, corresponding to angles diffracted around of $\theta = 60$ ° (corresponding to the diffracted energy along the oblique incidence angle $\theta_i = 30$ °) and for the angles 58 °, 57 ° and 64 °.

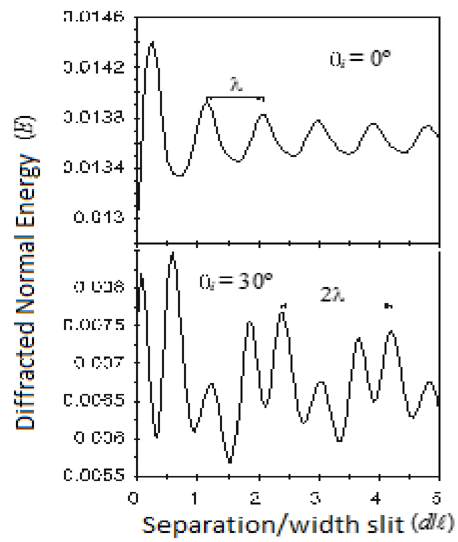

Figure 4. Energy diffracted in the direction normal to the standard E_i to Hermite-Gauss beam, depending on the spacing d/ℓ screen. For the fundamental mode $m = 0$, at normal incidence and oblique incidence of 30°, with λ/ℓ=0.9, L/ℓ=500/$\sqrt{2}$, h/ℓ =1, y b/ℓ=0.5.

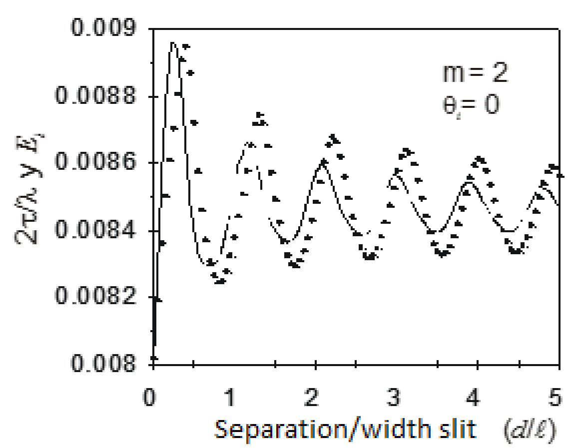

Figure 5. Energy diffracted in the direction normal to the E_i (solid line) Hermite-Gauss beam, thus $m = 2$ and $2\tau/\lambda$ property (dashed line), in function of the spacing d/ℓ. Same parameters of Fig. 3.

Energy analyzing energy diffracted E_i around for $m = 0$ at normal incidence and oblique incidence of 30 ° as also carried out (data not shown) found similar patterns for mode $m = 2$ (see Fig. 6), the energy diffracted around the energy as E_i a function of the spacing d, decay to zero.

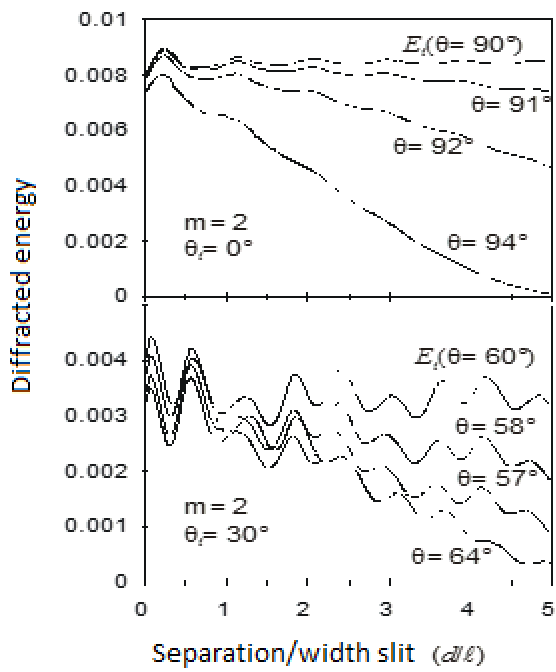

Figure 6. Energy diffracted around energy $E_i(\theta = \theta i)$ Hermite-Gauss beam, for the $m = 2$ mode according to the distance between slits d/ℓ. Same parameters of Fig. 4.

Conclusion.

Present a more rigorous theory of diffraction for the oblique incidence beam Hermite-Gaussian (HG) on a screen of thickness h with wide slits separating slits ℓ and d. In the case of TM(S) polarization and wavelengths in the vector region $\frac{\lambda}{\ell} > 0.2$, we have found that the diffracted along the direction of the incident beam energy has oscillations period λ as a function of the spacing d for modes $m = 0$ and 2, for the period 2λ at 30 ° oblique incidence. Finally, we note that the energy E_i has special characteristics compared diffracted energies in other directions and found numerically that ownership of scalar diffraction ($\lambda/\ell < 0.2$) given by $E_i = 2\tau/\lambda$ is no longer valid in this region ($\lambda/\ell > 0.2$).

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