Belyaeva, T.L.; Demyanova, A.S.; Goncharov, S.A.; Ogloblin, A.A.
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Sociedad Mexicana de Física A.C.
Distrito Federal, México

Disponible en: http://www.redalyc.org/articulo.oa?id=57030350005
Do exotic alpha-cluster states in $^{12}$C show signatures of “alpha-condensate” structure? Analysis of recent data on the $\alpha$ -particle inelastic scattering

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Received el 17 de febrero de 2009; aceptado el 7 de junio de 2009.

The diffraction model, DWBA and the Coupled Reaction Channels analysis of the novel data of $\alpha + ^{12}$C elastic and inelastic (to the states $4.44, 7.65$ and $9.64$ MeV) scattering in full angular range at an incident energy of $110$ MeV is presented. The diffraction radii for the ground and the first excited (4.44 MeV) states are found to be equal. The diffraction radii for the 7.65 and 9.64 MeV states are enhanced by 0.5–0.8 fm. This result shows that the radius of the Hoyle’s $0^+_2$, 7.65 MeV state in $^{12}$C is larger by a factor of $\sim 1.2 \cdot 1.3$ than that of the ground state. It is demonstrated that the direct transfer of $^8$Be dominates at large angles in all four reactions reported here and that the relative angular momentum $L = 0$ corresponding to the transfer of $^8$Be in its ground state $0^+$ has predominant probability for the $0^+_2$ state in comparison with the ground state of $^{12}$C. Evidence of existence of some features of alpha-condensed structure of the Hoyle’s $0^+_2$ state in $^{12}$C was obtained, particularly, its enhanced radius and large contribution of alpha-particle configuration with $L = 0$.

Keywords: Diffraction model, $\alpha$-particle + $^{12}$C elastic and inelastic scattering, diffraction radii of the alpha-cluster states in $^{12}$C, the radius of the 7.65 MeV 0$^+$ Hoyle state in $^{12}$C, alpha - condensate structure in $^{12}$C.

Presentamos el análisis de los nuevos datos de la dispersión elástica y inelástica (a los estados 4.44, 7.65 y 9.64 MeV) de las partículas alfa con la energía 110 MeV en $^{12}$C en los modelos de difracción, de los canales acolectados y DWBA para el intervalo completo de ángulos. Los radios de difracción para los estados básico y primero excitado (4.44 MeV) se encuentran iguales. Los radios de difracción para los estados con $E_x = 7.65$ y 9.64 MeV se incrementan en 0.5 - 0.8 fm. Este resultado muestra que el radio del estado $0^+_2$, 7.65 MeV de Hoyle en $^{12}$C es más grande por un factor de $1.2 \cdot 1.3$ que el radio del estado básico. Se demuestra que la transferencia directa de $^8$Be domina en los ángulos grandes en las cuatro reacciones presentadas aquí, y que el momento angular relativo, $L = 0$, correspondiente a la transferencia de $^8$Be en su estado básico $0^+$ tiene la probabilidad predominante para el estado $0^+_2$ en comparación con el estado básico de $^{12}$C.

Descriptores: Modelos ópticos y de difracción; dispersion elastic e inelástica; fuerzas en sistemas de hadrones e interacciones efectivas.

PACS: 24.10.Ht; 25.55.Ci; 27.20.+n; 21.60.Gx

1. Introduction

The idea of clustering in the light nuclei proposed more than 50 years ago received a new impetus due to a recent conjecture of possible existence of an $\alpha$ - particle Bose-Einstein condensation ($\alpha$BEC) in $\alpha$-cluster nuclei [1]. Numerous calculations based on cluster models, as well as microscopic single-particle Nilsson-Strutinsky model, demonstrated that different types of clustering can appear in a nucleus in the region of excitation energy where it becomes energetically favorable to break up into those cluster fragments. The energy surfaces for alpha-cluster nuclei reveal a few minima with different deformations and the density distributions of these correspond to the triangular or chain configurations for $^{12}$C, tetrahedral, kite and chain ones for $^{24}$Mg [2]. $^{12}$C nucleus is considered as one of the best examples of alpha clustering phenomenon. There exist some theoretical predictions that the second $0^+$, 7.65 MeV state may be considered as the best candidate of the $\alpha$ -condensed state [1].

On the other hand, the excited states of light nuclei lying near or above the particle-decay threshold play an important role in the nucleosynthesis in stars. As early as in 1954, Hoyle showed [3] that the observed amount of carbon in the cosmos could be made in stars only if there was an excited state in carbon with a particular spin and parity, $0^+$, and a particular energy, about 7.6 MeV that corresponds to the fusion of three $\alpha$-particles. That state, soon found experimentally [4], is located slightly above (0.38 MeV) the decay threshold of $^{12}$C to three $\alpha$-particles, at $E = 7.65$ MeV with $\Gamma = 8.5 \pm 1.0$ eV. The properties of the Hoyle state in $^{12}$C determine the ratio of carbon to oxygen formed in the stellar helium burning process that strongly affects the future evolution of the star. Stars that have exhausted the hydrogen fuel in their cores by converting it to helium, then convert that helium to carbon and oxygen by a sequence of two reactions: three $\alpha$ - particles fusion to form $^{12}$C, and, sometimes, by the capture of a fourth $\alpha$ - particle, therefore forming $^{16}$O.

In 1956 Morinaga [5] suggested that the Hoyle state has a linear chain structure of three $\alpha$-particles. Later, microscopic calculations based on the Bloch-Brink microscopic alpha-particle model [6] confirmed the $\alpha$ - clustering, but indicated that the 7.65 MeV state should rather be considered as a Bose gas of alpha-particles than the chain-like state suggested by Morinaga. The hypothesis of $\alpha$BEC put forward by Tohsaki et al. in 2001 [1], includes the following features:

(a) In A=4n nuclei one could expect the existence of excited states of dilute density composed by weakly interacting $\alpha$ – particles, which could be considered as $n\alpha$ – cluster condensed states ($n=2,3,4,\ldots$);
(b) Corresponding nuclear states are located closely to the thresholds of complete dissociation to \( \alpha \)-particles: \( A \to n\alpha \);

(c) Almost unperturbed alpha-particles might have zero relative angular momentum \( L=0 \), that corresponds to a \( \delta \)-function like peak around \( k=0 \) in the lineal momentum distribution.

(d) Their radii are estimated to be larger by a factor of 1.4 - 1.7 than those for the ground state [6];

(e) Such \( \alpha \)-condensed states might be a general feature in \( N=Z \) nuclei.

The concept of Bose-Einstein condensation (BEC) is well-known for macroscopic systems of cold atoms and molecules in the external fields. Is it possible to extent this concept to the self-bound nuclear systems with a small number of particles? Two natural questions arise: Are the properties of the excited nuclear state mentioned above (a)-(d) sufficient to declare this state as an \( \alpha \)-condensed one? How can we test experimentally the condensate properties of high-lying states?

It is not easy to answer these questions. What is clear is that the principal feature of the atomic BEC, the coherence of all individual wave functions of atoms in the external field and the compression of the total wave function due to nonlinear interaction, is not realized in the case of finite nuclei.

Nevertheless, coherent effects in finite nuclei manifest themselves by many ways and usually are expressed in terms of different correlations. These correlations are responsible for creation of the nucleon associations with their energetic advantages. Nucleon-nucleon correlations leading to pairing effect, as well as the \( \alpha \)-type correlations are important in the light, and \( \alpha \)-conjugate nuclei, and heavy nuclei [7].

In this sense, it is argued in Ref. 8 that the concept of a nuclear condensate refers to well-known cluster states and no observable differences can be constructed to distinguish these alleged novel structures from ordinary cluster states. Thus, one might just talk about a "Bose gas", or a dilute systems of almost unperturbed alpha-clusters. Regardless of what are these states called, the idea of \( \alpha \)-BEC, first, has caused a rebirth of interest to the study of the structure of Hoyle and other near- and above-threshold states, and secondly, inspired us to think about how can we measure the radii of nuclei in the unbound states.

From the experimental point of view the main task of checking the condensed nature of a Hoyle state means verifying that at least:

(1) the 7.65 MeV state really has significantly enlarged dimensions, and

(2) the predominant configuration of alpha-clusters corresponds to their relative angular momentum \( L = 0 \). Ogloblin et al. [9] discussed several possible experimental approaches to study the properties of interest of the 7.65 MeV state in \( ^{12}\text{C} \).

Among them are:

(a) Shift of the positions of the rainbow minima in the inelastic scattering to this level (as it was originally proposed in Ref. 10);

(b) Extraction of the empirical inelastic form-factor from the \( \alpha \)– and \( ^3\text{He} \) scattering and comparing it with theoretical predictions;

(c) Getting information on the \( ^{8}\text{Be} \) transfer reaction form-factor. The authors of Ref. 9 came to the conclusion that new measurements of \( \alpha+^{12}\text{C} \) elastic and inelastic scattering at \( \approx 120 \text{ MeV} \) performed in full angular range are required.

As it was mentioned in Ref. 11, the most appropriate experiment of the elastic electron scattering on the Hoyle state is impossible because of its short lifetime (the half-life of the \( 0^+, 7.65 \text{ MeV} \) state is \( \approx 2 \times 10^{-16} \text{ sec.} \)).

These measurements of the elastic and inelastic \( \alpha \)-particle scattering on \( ^{12}\text{C} \) at 60 MeV (Kurchatov Institute cyclotron, Russia) and at 110 MeV (Jyvaskyla University cyclotron, Finland) were realized at the end of 2006. The differential cross sections of the \( \alpha ^{+12}\text{C} \) scattering were obtained in large angular range \((\theta_{lab} = 10-170^\circ)\) with formation of four lower states (ground state, 4.44, 7.65, and 9.64 MeV) [12]. Despite of the existence of a large number of \( \alpha + ^{12}\text{C} \) scattering data, not all of them cover the full angular range.

Besides the Hoyle state, a possible \( \alpha \)-condensed structure is predicted for the states in \( ^{16}\text{O} \), which are located around the threshold of complete dissociation to 4 \( \alpha \)-particles (the energy of dissociation \( ^{16}\text{O} \to 4\alpha \) is 14.4 MeV). Wakasa et al. [13] proposed that the state 13.6 MeV (its RMS radius is estimated to be \( \langle r^2 \rangle^{1/2} = 4.3 \text{ fm} \), that is larger by a factor of 1.64 in comparison with the ground state) has a possible \( \alpha \)-BEC structure. Recent calculations of Funaki et al. [14] demonstrated the enlarged radius (~4.0 fm) of the state 13.6 MeV, but considered the above-threshold state 15.05 MeV \(( \langle r^2 \rangle^{1/2} > 5 \text{ fm} \) as the most probable candidate to have an \( \alpha \)BEC structure. The diffraction analysis of the recent elastic and inelastic \( \alpha + ^{16}\text{O} \) data is in progress.

The main task of the present work is to analyze the recent elastic and inelastic \( \alpha + ^{12}\text{C} \) data and to clarify what signatures of "alpha-condensate" structure we really observe in \( ^{12}\text{C} \).

2. Elastic and inelastic scattering analysis

The experimental angular distributions of elastically and inelastically scattered \( \alpha \)-particles on \( ^{12}\text{C} \) at \( E_{lab} = 110 \text{ MeV} \), shown in Fig. 1, clearly reveal three characteristic regions:

1) small angle diffraction,

2) nuclear rainbow exponential slope between \( \sim 50-90^\circ \) in c.m. system, and
DO EXOTIC ALPHA-CLUSTER STATES IN $^{12}$C SHOW SIGNATURES OF “ALPHA-CONDESATE” STRUCTURE? ANALYSIS…

3) backward diffraction structures, which are expected to be the result of coherent superposition of scattering and $^3$Be transfer amplitudes.

As a first step for estimating the dimensions of the states, an analysis of the forward Fraunhofer diffraction in the framework of the strong absorption model of scattering was made [15]. It allowed the extraction of the scattering diffraction radii from the positions of the small angles minima and maxima not only for particle stable levels, but also of those located above the $\alpha$-decay thresholds. For elastic scattering the sum of “real” radii differs from the diffraction radius by some value $\Delta$, which is determined by the details of interaction and the matter radial distributions of the colliding nuclei:

$$R_{dt,f} = R_{rms}(^{12}_C g.s.) + R_{rms}(^4_He) + \Delta.$$  

The analysis of the positions of the 1st, 2nd and in some cases 3rd minima of the angular distributions of $\alpha + ^{12}_C$ elastic and inelastic scattering to the four lowest states measured in Ref. 12, together with the published data [16-20] at 104, 139, 166, 172.5 and 240 MeV, as well as the same analysis for $^3$He + $^{12}_C$ scattering measured at 72 MeV [21], allowed us to make the following conclusions:

(a) The diffraction radii extracted from the analysis of the elastic and inelastic $\alpha + ^{12}_C$ (Fig.2) and $^3$He + $^{12}_C$ scattering decrease with the energy, and their energy dependences are described approximately by quasi-linear functions (the solid curves in Fig.2);

(b) The $R_{dif}$ values extracted from the elastic scattering are related directly with the known values of $R_{rms}$ of the nuclei in the ground states (the rms radii of the charge distributions of $^3$He, $^4$He, $^{12}_C$ g.s. and $^{16}_O$ g.s. were taken as 1.66, 1.47, 2.34 and 2.625 correspondingly), so the averaged differences $\Delta$ for the elastic scattering can be calculated directly. A quite systematic quasi-linear decrease of $\Delta$ as a function of energy is observed for different projectile – target combination in wide energy interval.

(c) In the case of the inelastic scattering the initial and final states are different and $\Delta(\text{inel})$ values differ, in general, from $\Delta(\text{el})$ values. As a first step for estimating the radii of the excited states we assumed that the same value of $\Delta$ is valid for both elastic and inelastic scattering and that way have got rms radii of the excited states from the corresponding diffraction radii in accordance with Eq.(1) (See Table I).

(d) The diffraction radii of the ground state and the first $2^+, 4.44$ MeV excited state appear to be approximately the same. The diffraction radius of $0^+_g, 7.65$ MeV level is larger by $\approx 0.6$ fm, and this difference does not depend on the energy. Thus, the radius of the Hoyle state is really enhanced in comparison with the lower lying levels and close to the predictions [6].

(e) The radius of $3^-, 9.64$ MeV state is of the same magnitude as the radius of the Hoyle state. This fact can be interpreted rather with the very fact of locating these states above the threshold of $\alpha$-decay.

\begin{table}[h]
\centering
\caption{\label{tab1} $R_{rms}$ radii of $^{12}_C$ in the excited states obtained from the $\alpha + ^{12}_C$ and $^3$He + $^{12}_C$ scattering data}
\begin{tabular}{|c|c|c|}
\hline
$J^p, E^*$ & $\alpha + ^{12}_C$ & $^3$He + $^{12}_C$ \\
\hline
$2^+, 4.44$ & 2.38±0.07 & 2.38±0.04 \\
$0^+, 7.65$ & 2.91±0.03 & 2.90±0.06 \\
$3^-, 9.64$ & 3.00±0.06 & 3.10±0.09 \\
\hline
\end{tabular}
\end{table}
Table II. Optical-potential parameters for $\alpha + ^{12}C$ channels.

<table>
<thead>
<tr>
<th>$J^p, E^*$</th>
<th>$V_0$</th>
<th>$R_0$</th>
<th>$a_0$</th>
<th>$W$</th>
<th>$R_W$</th>
<th>$a_W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MeV</td>
<td>fm</td>
<td>fm</td>
<td>fm</td>
<td>MeV</td>
<td>fm</td>
<td>fm</td>
</tr>
<tr>
<td>2$^+$, 4.44</td>
<td>100.0</td>
<td>3.295</td>
<td>0.800</td>
<td>21.00</td>
<td>4.290</td>
<td>0.453</td>
</tr>
<tr>
<td>3$^-$, 9.64</td>
<td>115.0</td>
<td>2.71</td>
<td>0.90</td>
<td>15.00</td>
<td>4.290</td>
<td>0.750</td>
</tr>
</tbody>
</table>

We performed an analysis of $\alpha + ^{12}C$ elastic scattering data under the condition that the optical model potentials reproduce correctly the energy dependence of Airy minima positions. The analysis of all available data in energy range $E_{lab} = 41 - 240$ MeV [15-19] including our new one at 110 and 60 MeV [12] was performed using the semi-microscopic dispersive optical model (SMDOM) (see, for instance, [22] and references therein), where the optical potential (OP)

$$ U = V_F + V_p + iW + V_C, $$

is written as a sum of the mean-field real potential $V_F$

$$ V_F(r, E) = V^F(r) + [1 + \varphi(E)] W^{(SNKE)}(r, E), $$

which can be calculated in folding model accounting exchange effects by corrected SNKE approximation, the complex dynamic polarization potential (DPP) $V_p + iW$ and the Coulomb potential $V_C$. The real and imaginary parts of the DPP of the Woods-Saxon radial shape are constructed phenomenologically by fitting the geometrical parameters (radii $r_S$, $r_D$ and diffusions $a_S$, $a_D$) which are supposed to be energy independent, and energy dependent potential strengths ($V_S$, $V_D$, $W_S$, $W_D$). In general, the mean-field real potential $V_F$ and the complex DPP are connected by dispersive relations, as well as the integral characteristics of OPs, real $J_V(E)$ and imaginary $J_W(E)$ volume integrals. The Coulomb interaction is represented by the potential of a uniformly charged sphere with a radius $r_C = 1.1$ fm. The details of the formalism are presented in Ref. 22. Our analysis shows that by using the universal geometric parameters $r_S = 1.1$ fm, $a_S = 0.46$ fm, $r_D = 0.54$ fm, $a_D = 0.78$ fm and adjusting the potential strengths, we are able to obtain a good description of the angular distributions of $\alpha + ^{12}C$ elastic scattering and reaction cross sections at wide energy range from 41 to 240 MeV.

For the $2^+$- and $3^-$-final states de exact SMDOM optical potentials were approximated by the standard 6-parametric complex nuclear OP of the form

$$ V(r) = -V_0 f_0(r) - iW f_W(r), $$

$$ f_i(r) = \left\{ 1 + \exp\left[\frac{(r - R_i)/a_i}{3}\right] \right\}^{-1}, \quad R_i = r_i (A^1_p + A^1_f) $$

presented in Table II.

The obtained SMDOM optical potential at $E_0 = 110$ MeV has the following parameters: $\varphi = 0\.09$, $-V_p = 5\.35$ MeV, $-V_p = -19\,2$ MeV, $-W_S = 6\,60$ MeV, $-W_D = 12\,8$ MeV, $-J_V = 344$ MeVファイマ, $-J_W = 116$ MeVファイマ³. It was applied.
to calculate the input channel of the reactions. The same SM-DOM potential was used for the calculations of the $0^+_2$ - final channel.

The differential cross sections for the inelastic scattering were calculated in the DWBA using the DWUCK4 code [23]. To construct the inelastic form-factors we used a phenomenological model with different parametrization of the radial shapes of form-factors. The first derivative of the Woods-Saxon shape was used to describe form-factors of the $2^+$ - and $3^-$ - states, and the second derivative was applied to describe the form-factor of the $0^+_2$ - state. The latter is more adequate to the shape of the nuclear transition density obtained in Ref. 6. The calculated angular distributions shown in Figs. 1, 3, and 4 by dotted curves reproduce quite well the experimental data for $\theta_{\text{c.m.}} < 100^\circ$. The parameters of the form-factors are shown in Table III.

3. Large angle scattering and $^8$Be transfer

The $\alpha$ - particle scattering at 110 MeV is characterized by pronounced enhancement and strong oscillations at large angles. We have already mentioned above that at $E_{\alpha}=110$ MeV the Fraunhofer diffraction is well divided from the large angle oscillations by a wide region of rainbow scattering. In fact, these oscillations are extended down to $\sim 90^\circ$ in c.m. system and have quite regular character (see Fig.1). The experimental cross sections at large angles exceed those calculated in optical model and DWBA (dotted lines in Figs.1,3,4) by a factor of $10^4$, thus, we can propose the presence of some non-potential mechanism, most probably, a $^8$Be direct transfer.

<table>
<thead>
<tr>
<th>Table IV. Parameters of the potentials of $\alpha + ^8$Be interaction $V(r) = V[1 + \exp [(r - R_v)/a_v]]$</th>
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</thead>
<tbody>
<tr>
<td>$L^+_\alpha, E^*$</td>
</tr>
<tr>
<td>$0^+$, g.s.</td>
</tr>
<tr>
<td>$2^+$, 4.44</td>
</tr>
<tr>
<td>$0^+$, 7.65</td>
</tr>
<tr>
<td>$3^-$, 9.64</td>
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</table>

$^1 R_V = r_{0V}(A_1^{1/3} + A_2^{1/3}), A_1 = ^4$He, $A_2 = ^8$Be

<table>
<thead>
<tr>
<th>Table V. Reduced width amplitudes $\Theta_{L_1 I_1 I_X}(^{12}$C$\rightarrow ^8$Be$+\alpha)$ (abs.values) obtained from comparison between experiment data and calculations.</th>
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</thead>
<tbody>
<tr>
<td>$L_{12}$C, $E^*$</td>
</tr>
<tr>
<td>$0^+$, g.s.</td>
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<tr>
<td>$0^+$, 7.65</td>
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We calculated the differential cross sections of the $^{12}$C$(\alpha, \alpha')^{12}$C$^*$ reaction for the different excited states in $^{12}$C under the assumption that the direct transfer mechanism or heavy stripping of $^8$Be cluster takes place. The calculations were carried out by the Coupled Reaction Channels Method (CRC ) [24]. The computer code FRESCO developed by I. J. Thompson [25] was used for the calculations. We used the same optical potentials (Table II) as for the scattering calculations. Normalized single-particle $\alpha + ^8$Be wave functions generated by a Woods-Saxon potential (see Table IV) describe bound and “weakly” bound states of relative motion of $\alpha + ^8$Be in $^{12}$C($J^p, E^*$).

Let us consider a nuclear reaction $A(\alpha, b)B$, which is realized via the heavy cluster transfer mechanism,

$$a(I_a) + A(I_A) \rightarrow a + [X(I_X) + b] \rightarrow [a + X(I_X)] + b \rightarrow B(I_f) + b(I_b).$$

We use a standard angular-momentum coupling scheme

$$l = I_1 + I_2 = L_1 + L_2,$$

$$I_f = I_1 + I_a, I_A = I_2 + I_b,$$

$$I_1 = I_X + L_1, I_2 = I_X + L_2,$$ (4)

where $l$ is the transferred angular momentum, $L_1$ and $L_2$ are orbital angular momenta of relative motion of the clusters $a + X$ and $b + X$ in the nuclei $B$ and $A$, respectively, and $I_1$ and $I_2$ are total transferred angular momenta.

The calculated differential cross sections show a strong sensitivity to the structure of the given excited state in $^{12}$C, that is, to its cluster wave function and to the relative weight of the various $[\alpha \otimes ^8$Be$](I_X)$ configurations. The latter is characterized by the reduced width amplitudes (RWA) $\Theta_{L_1 I_1 I_X}(^{12}$C$\rightarrow ^8$Be$+\alpha)$, which we estimated by a comparison of the experimental and calculated differential cross sections. The extracted values of RWA for the $0^+_2$ and $0^+_3$ states are shown in Table V.

It is important to note that the calculations of the differential cross sections in CRC method include the coherent sums over total momentum $I_X$ of the transferred nucleus and over the relation angular momenta $L_1$ and $L_2$. We took into account a direct transfer of the $^8$Be$^*$ cluster in the $g.s.$ and in the first excited $2^+$ and $4^+$ states. Contributions of the different $\alpha \otimes ^8$Be$^*$ configurations are indicated by thin curves in Fig. 3. Their coherent sum indicating the effects of the coupling of different cluster configurations is denoted by the dashed curves.

Our analysis shows that a direct transfer of the $^8$Be$^*$ produces a major contribution to the cross sections at large angles as for the $0^+, g.s.$ and for the $2^+$, 4.44 MeV, $0^+$, 7.65 MeV and $3^-$, 9.65 MeV states and allows us to reproduce their oscillated structure.

Let us compare the relative contributions of different $[\alpha \otimes ^8$Be$](I_X)$ configurations for the $0^+_1$ and $0^+_2$
states (see Table V). The ratio of the RWAs with relative angular momenta \( L_1 = L_2 = 0 : 2 : 4 \) is \( 1 : 1.6 : 1.6 \) for the \( 0^+_2 \) state in comparison with \( 1 : 0.64 : 0.5 \) for the \( 0^+_2 \) state. This enhancement of a contribution of zero-momentum component, together with the enlarged radius of the state, can be considered as a signature of an enlarged independence of \( \alpha \)-clusters and certain diluteness of the density distribution in the \( 0^+_2 \) Hoyle state. These results are comparable with the recent calculations [26] of the \( 3\alpha \) – cluster structure of excitations states in \( ^{12}\text{C} \) based on the Complex Scaling Method, which indicated that the amplitude ratio for \( L_1 = L_2 = 0 : 2 : 4 \) is \( 1.0 : 1.04 : 0.83 \) for the \( 0^+_1 \) state and \( 1.0 : 0.4 : 0.32 \) for the \( 0^+_2 \) state. Our analysis reveals an important role of the large momentum component, \( L_1 = L_2 = 4 \), which forms a diffraction picture at large angles. It should also be noted, that the interference between different configurations results in rearrangement of their contributions.

Also, it is interesting to compare our results with the calculations realized in the semi-microscopic \( 3\alpha \) cluster model [27], where the authors analyzed the characteristic structure of the \( 0^+ \) states studying the single-\( \alpha \) orbital behavior in these states as a function of the nuclear radius \( R_N \). It was found that for the ground state with nuclear radius \( R_N=2.44 \) fm, the occupation probability of the \( \alpha \) particles is distributed among the three \( L=0,2,4 \) orbits, with each of 30%. While the \( 0^+_2 \) state appeared to be a dilute \( 3\alpha \) system characterized by the nuclear radius as large as 4.3 fm. About 70% of the occupation probability of the single-\( \alpha \) orbits in this case concentrates on a single S-orbit (\( L=0 \)). The prominent peak in the \( k < 1 \text{fm}^{-1} \) region in the momentum distribution reflects a domination of the S-wave motion.

In our calculations, in fact, the ratio of the spectroscopic factors \( S(L) \) characterizes probability of the \( \alpha \)-particle \( ^8\text{Be} \) relative motion with certain orbital momentum. From Table V we can calculate that \( S(0):S(2):S(4) = 16% : 42% : 42% \) for the ground \( 0^+_2 \) state versus \( 60% : 25% : 25% \) for the \( 0^+_2 \) Hoyle state. Though this ratio, as well as the estimated r.m.s. radius of the Hoyle state, are smaller than the predicted by Ref. 27, our results demonstrate the same tendency to the predominant probability of the relative angular momentum \( L=0 \) and enlarged radius of the \( 0^+_2 \) state in \( ^{12}\text{C} \) in comparison with the ground state.

We compare the calculated inelastic scattering cross sections with experimental data [12] for the \( 2^+ \), 4.44 MeV and \( 3^-, 9.65 \) MeV final states. For the \( I^+_1 = 2^+ \) state, the coupling scheme (5) allows the following values of angular momenta: \( I_1 = I_f = 2, I_X = L_1 = L_2 = 0, 2, 4, I_X + L_1 = 2 \). The calculations display almost positive interference of all configurations with a slightly enhanced contribution of the maximum momentum component, \( L_1 = L_2 = 4 \).

The negative-parity excited states in \( ^{12}\text{C} \) are observed above the 3\( \alpha \)threshold, and are considered to have \( \alpha \) – cluster structure. Recently, Kurokawa and Katō [26] have demonstrated that the \( 3^- \), 9.65 MeV state may correspond to the head of the \( K^r = 3^- \) rotational band with the equilateral triangular \( 3\alpha \) spatial configuration. Corresponding values of angular momenta are: \( I_1 = I_f = 3, I_X = L_2 = 0, 2, 4, L_1 = 1, 3, 5, I_X + L_1 = 3 \). The direct transfer of the \( ^8\text{Be} \) into \( ^{12}\text{C}(3^-) \) state is forbidden in the shell model by momentum and parity conservation laws. Nevertheless, the pronounced enhancement and strong oscillations at large angles observed in the data [12] strongly manifest the exotic \( 3\alpha \) structure of this state. The calculations show a positive interference of all configurations with the same RWA values.

4. Conclusions

The analysis of the differential cross-sections of \( ^{12}\text{C} + \alpha \) elastic and inelastic scattering at 110 MeV in the angular range of 10 to 175° was performed. The semi-microscopic dispersive optical model and DWBA calculations reproduce well the elastic and inelastic scattering cross-sections in the forward hemisphere. The cross-sections at backward angles are calculated in the coupled reaction channel model assuming a direct transfer of the \( ^8\text{Be} \). In the framework of the diffraction model we obtained that the diffraction radii of the states \( 0^+_2 \), \( 7.65 \) MeV and \( 3^- \), \( 9.64 \) MeV located above the threshold \( ^{12}\text{C} \rightarrow 3\alpha \) are significantly larger (by \( \sim 0.6 \) fm) than those below it, and found that the radius of the \( 0^+_2 \) Hoyle state in \( ^{12}\text{C} \) is larger by a factor of \( \sim 1.2-1.3 \) than that of the ground state. Though this value is smaller than the predicted by the theory, it can be considered as an argument in favour of a condensed structure of the Hoyle state. The direct \( ^8\text{Be} \) transfer calculations showed that the cluster configuration with the angular momentum of relative motion of alpha-particles \( L = 0 \) is three times larger than that in the ground state and dominates in Hoyle state, indicating its possible \( \alpha \)BEC structure. On the other hand, contributions of more complicated configurations with \( L=2 \) and \( L=4 \) are quite noticeable. To conclude, some important features inherent to an hypothetical \( \alpha \)BEC structure were identified in \( 0^+_2 \), \( 7.65 \) MeV state of \( ^{12}\text{C} \). However, its real cluster structure seems to be rather complicated and cannot be converged to a simple single model.

Acknowledgements

This work is partially supported by UAEM project 2428/2007.


