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“CONDICIONES RELACIONADAS CON LA PSEUDOCONTRACTIBILIDAD Y SELECTIBILIDAD”

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DOCTOR EN CIENCIAS

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CONDICIONES RELACIONADAS CON LA PSEUDOCONTRACTIBILIDAD Y LA SELECTIBILIDAD

Tesis por artículos especializados

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Índice general

Capítulo 1

Anteproyecto de investigación

ANTEPROYECTO DE INVESTIGACIÓN DE DOCTORADO EN CIENCIAS (MATEMÁTICAS)

TÍTULO
Condiciones Relacionadas con la Pseudocontractibilidad y Selectibilidad
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INTRODUCCIÓN

La Teoría de Continuos es una rama de la topología que estudia las propiedades de los espacios métricos, compactos y conexos.

La primera definición de continuo (que varía a la actual) fué dada por G. Cantor un par de decadas antes de la llegada del siglo XX. Entre los años 10's y 20's del siglo XX se desarrolla la Teoría de Hiperespacios de Continuos la cual estudia propiedades de ciertos subconjuntos del conjunto potencia de X.

En México, durante los últimos 30 años, este tipo de espacios se han estudiado ampliamente con gran éxito. En el Estado de México se cuenta con al menos 5 espacialistas, quienes han contribuido tanto en producción académica como en la formación de recursos humanos dentro de esta área de estudio.

Dos de los temas que han inquietado tanto a investigadores nacionales como extranjeros son: la contractibilidad y la selectibilidad, algunas propiedades topológicas que están relacionadas con estos temas son los siguientes: tener la propiedad de la intersección doblada, ser uniformemente arcoconexo, contener Q-puntos, ser de tipo N(ver [18]), tener R^i continuos o tener conjuntos homotópicamente fijos (ver [19]) entre otros. particularmente si el espacio topológico es abanico se sabe que tener Q-puntos, no ser suave a pares o no tener la propiedad de intersección doblada, son propiedades que no permiten que un espacio sea contráctil (ver [11]). Por otra parte se sabe que si un espacio es selectible tiene la propiedad de intersección doblada. Se sabe que la propiedad de intersección doblada no caracteriza a los dendroides selectibles con más de un punto de ramificación, sin embargo aún no se sabe si la propiedad de intersección doblada caracteriza a cierta clase de abanicos.
La siguiente pregunta abierta relaciona los conceptos de selectibilidad y contractibilidad:
Pregunta: ¿Si X es un abanico contraćtil (pseudo-contráctil), entonces será selectible?

Por lo que es imposible estudiar la selectibilidad sin echarle un vistazo a la contractibilidad y viceversa.

Otro concepto que ha sido poco estudiado y que aparecen en la literatura, es el concepto de pseudocontractibilidad, este concepto generaliza el concepto de contractibilidad.
En este proyecto se investigará, entre otras cosas, si alguna o algunas de las propiedades mencionadas anteriormente impiden que un espacio sea pseudocontráctil o selectible.

1. Definiciones básicas

Sean $f, f' : X \to Y$ funciones continuas. Una función continua $H : X \times [0,1] \to Y$, se dice que es una homotopía entre f y f' si $H(x,0) = f(x)$ y $H(x,1) = f'(x)$ para $x \in X$. Si sucede lo anterior podemos decir que f y f' son homotópicas. Se dice que un espacio X es contraíble si existe una homotopía entre la función identidad y una función constante en X.

Una función continua $H : X \times C \to Y$, donde C es un continuo, se dice que es una pseudohomotopía entre f y f' si existen puntos $a, b \in C$ tal que $H(x,a) = f(x)$ and $H(x,b) = f'(x)$ para $x \in X$, también se puede decir que f y f' son pseudohomotópicas. Un espacio es llamado pseudocontraíble si existe una pseudohomotopía entre la función identidad y una función constante en X.

Dado un espacio topológico X y un subconjunto cerrado$A$ de X, una retracción de X sobre A, es una función continua $r : X \to A$ tal que $r|_A = id_A$. Se sabe que una selección s es una retracción de $C(X)$ sobre X en donde $s(A) \in A$ para toda $A \in C(X)$.

2. Antecedentes

Desde antes de los años 70's se ha buscado caracterizar a los continuos contraíbles como se puede ver en [10], [11], [13], [18], [19], [20] entre otras. Por ejemplo en [19] y en [13] se dá una caracterización de abanicos contraíbles.

Por otro lado la cantidad de resultados que se conocen de la pseudocontractibilidad son menores, pero datan de la década de los 70's. Investigadores como W. J, Charatonik han hablado en diferente congresos en relación a este tema.

Algunos resultados que se conocen son los siguientes: todo continuo pseudocontraíble tiene forma trivial, H. Katsuura fue el primero en probar en [25] que la curva sen$\frac{1}{x}$ no es pseudocontraíble.

Posteriormente en [21] se probó que el único continuo encadenable que es pseudocontraíble es el arco, particularmente el pseudoarco es otro ejemplo de un continuo que no es pseudocontraíble.

Otro resultado conocido es el siguiente:
Teorema: Sea Y un continuo indescomponible, tal que cada composante es arcoconexa. Si Z es un continuo tal que tiene una arco-componente propia no degenerada entonces Z no es pseudocontraíble relativo a Y.
Como consecuencia de este resultado se tiene que sen$\frac{1}{x}$ no es pseudocontraíble relativo a Y donde Y tiene las propiedades antes mencionadas.
Como la curva sen$\frac{1}{x}$ es de tipo N, conjeturamos que cualquier curva de tipo N no

es pseudocontraíble con respecto a Y donde Y tiene las propiedades del teorema.

Con relación a la selectibilidad, se sabe que si X es un dendroide selectible (o contráctil) entonces el dendroide es uniformemente arcoconexo. Por otra parte, Maćkowiak en [14], prueba que ser selectible implica que el dendroide tiene la propiedad de intersección doblada. En el mismo artículo el autor prueba que la implicación inversa no es cierta, para ello, muestra un dendroide con dos puntos de ramificación. Por este motivo J. J. Charatonik, W. J. Charatonik y S. Miklos [9, Pregunta 14.6], preguntan lo siguiente: ¿Existe un abanico (dendroide con un sólo punto de ramificación), que no sea selectible y que tenga la propiedad de intersección doblada?. En [3] se responde afirmativamente a esta pregunta, es decir, existe un abanico con la propiedad de intersección doblada el cual no es selectible.

Dado que el ejemplo mostrado en [3] no es plano y no es localmente conexo en el vértice, planteamos la siguiente conjetura:
Todo abanico localmente conexo en el vértice o plano con la propiedad de intersección doblada es selectible.

Una pregunta (mencionada ya) que está ampliamente relacionada con la conjetura anterior es la siguiente:
Pregunta: ¿Si X es un abanico contraíble (pseudo-contráctil), entonces será selectible?

3. PROBLEMAS DE INVESTIGACIÓN

(1) El primer problema es determinar cuales de las propiedades mencionadas anteriormente no permiten la pseudocontractibilidad. Adicionalmente se buscarán condiciones necesarias y suficientes para que un espacio o una familia de espacios con cierta(s) propiedad(es) sea(n) pseudocontráctil(es), comenzando el estudio para el caso particular en el que el espacio (continuo) sea de tipo N.

(2) El segundo problema de estudio es ver si la conjetura mencionada anteriormente es cierta, para de esta forma determinar si la pregunta antes planteada es cierta o no. Nosotros creemos que es cierta. Por otra parte queremos determinar que clases de funciones entre dendroides o abanicos preservan o no preservan selectibilidad (no selectibilidad) y/o la pseudo-contractibilidad (no pseudo-contractibilidad). Este problema aplicado a la contractibilidad y selectibilidad ya ha sido ampliamente estudiado en [4], [5], [8], [14] y [16]. Por ejemplo en [3] los autores autor prueba que entre abanicos, ser selectible no es preservado bajo funciones confluentes-ligeras y ser no selectible, no es preservado bajo funciones confluentes-ligeras, abiertas y abiertas-ligeras. Nosotros queremos determinar si la selectibilidad y/o pseudo-contractibilidad es preservada bajo funciones abiertas o abiertas-ligeras en abanicos o en su defecto queremos determinar en que clase de abanicos la selectibilidad y/o pseudo-contractibilidad se preserva bajo las funciones antes mencionadas.

4. METODOLOGÍA

La metodología que seguiremos es la siguiente. Se realizará un revisión bibliográfica amplia de resultados ya conocidos relacionados a los problemas planteados, esto con el objetivo de revisar las técnicas usadas y determinar si estas son de ayuda para nuestro objetivo. En paralelo, la revisión de los mismos permitirán proveer de la herramienta adecuada para resolver los problemas planteados. Posteriormente se realizará una exposición de los mismos en un seminario con los tutores académicos. Finalmente se hará la investigación de los problemas planteados y se expondrán a los tutores acádemicos en un seminario para ser revisados.

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Capítulo 2

Artículo: Selectibility is not preserved under open light mappings between fans

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SELECTIBILITY IS NOT PRESERVED UNDER OPEN LIGHT MAPPINGS BETWEEN FANS

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AND FERNANDO OROZCO-ZITLI

Abstract. In this paper, we give an example of an open-light mapping between fans that does not preserve selectibility, which is an answer to the following question posed by Tadeusz Maćkowiak in *Continuous selections for* $C(X)$ [Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. **26** (1978), no. 6]: Does it follow that an open image of a selectible fan is selectible? Further, it is an answer to the following question posed by J. J. Charatonik, W. J. Charatonik, and S. Miklos in *Confluent mappings of fans* [Dissertationes Math. (Rozprawy Mat.) **301**, 1990]: Is selectibility invariant under mappings of fans that are (1) light and open, (2) open, (3) light and confluent?

1. Introduction

A *continuum* means a nonempty compact and connected metric space. A mapping is a continuous function. A continuum is said to be *hereditarily unicoherent* if the intersection of any two of its subcontinua is connected. An *arc* is understood as a homeomorphic image of a closed unit interval of the real line. If any two points of a space Z can be joined by an arc lying in Z, then Z is said to be *arcwise connected*.

A *dendroid* is defined as an arcwise connected and hereditarily unicoherent continuum. A point p of a dendroid X is called a *ramification point* of X (in the classical sense) if there exist three arcs emanating from

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p in X, with the intersection of each pair of them being just the singleton $\{p\}$. A *fan* means a dendroid having exactly one ramification point and this point is called its *top*.

Let X be a continuum. The hyperspace of all nonempty closed subsets of X is denoted by 2^X, the hyperspace of all subcontinua of X is denoted by $C(X)$ and the hyperspace of singletons is denoted by $F_1(X) = \{\{x\} : x \in X\}$, all equipped with the Hausdorff metric. Since $F_1(X)$ is homeomorphic to X, we may assume that $X \subset C(X)$.

By a *selection* for $C(X)$ we mean a mapping $\sigma : C(X) \to X$ such that $\sigma(A) \in A$ for each $A \in C(X)$. Then, X is said to be *selectible* provided that there is a selection for $C(X)$.

A mapping $g : X \longrightarrow Y$ between continua is said to be

- *monotone*, provided for each subcontinuum Q of Y, $g^{-1}(Q)$ is a continuum in X;
- *confluent*, provided for each subcontinuum Q of Y and each component C of $g^{-1}(Q)$, we have $g(C) = Q$;
- *light*, if the preimage $g^{-1}(y)$ is totally disconnected, for every $y \in Y$;
- *open*, if for any open set U in X, $g(U)$ is open in $f(X)$.

Let $p, q \in X$. We say that X is of *type N between p and q* if there exist in X an arc $A = \widehat{pq}$, two sequences of arcs $\{A_i\}_{i=1}^{\infty} = \{\widehat{p_ip'_i}\}_{i=1}^{\infty}$ and $\{B_i\}_{i=1}^{\infty} = \{\widehat{q_iq'_i}\}_{i=1}^{\infty}$, and points $p''_i \in B_i \setminus \{q_i, q'_i\}$ and $q''_i \in A_i \setminus \{p_i, p'_i\}$ for each $i \in \mathbb{N}$ (the symbol $\mathbb{N}$ stands for the set of all positive integers) such that

(1) $A = \mathrm{Lim} A_i = \mathrm{Lim} B_i$;
(2) $p = \lim p_i = \lim p'_i = \lim p''_i$;
(3) $q = \lim q_i = \lim q'_i = \lim q''_i$;
(4) each arc in X joining p_i and p'_i contains q''_i;
(5) each arc in X joining q_i and q'_i contains p''_i.

We say that a continuum X is of *type N* if X is of type N between two points in X.

The following definition was introduced by Tadeusz Maćkowiak in [5]. Let A be a subcontinuum of a continuum X and let $B \subset A$. We say that B is a *bend set* of A if there exist two sequences of subcontinua $\{A_n\}_{n=1}^{\infty}$ and $\{A'_n\}_{n=1}^{\infty}$ of X satisfying the following conditions:

(1) $A_n \cap A'_n \neq \emptyset$ for each $n \in \mathbb{N}$;
(2) $A = \mathrm{Lim} A_n = \mathrm{Lim} A'_n$;
(3) $B = \mathrm{Lim}(A_n \cap A'_n)$.

A continuum X is said to have the *bend intersection property* provided that, for each subcontinuum A of X, the intersection of all its bend sets is nonempty.

It is known that every selectible dendroid has the bend intersection property; see [5, Corollary, p. 548].

Is not difficult to prove that every dendroid of type N is not selectible.

The following question was formulated for the first time by Maćkowiak in [5, Problem, p. 550].

Question 1.1. Is selectibility between fans preserved under open mappings?

In this direction, J. J. Charatonik, W. J. Charatonik, and S. Miklos asked in [4] the following question.

Question 1.2. Is selectibility invariant under mappings of fans that are (1) open and light, (2) open, (3) confluent and light?

This question is a particular case of a more general one; see [4, Question 14.14].

Question 1.3. What kind of confluent mappings preserve selectibility (nonselectibility) of fans?

It is well known that the image of a selectible (nonselectible) fan under a monotone mapping need not be selectible (nonselectible); see [5, p. 549] and [4, Corollary 14.13].

In [2, examples 2.1 and 2.2] the authors gave examples to show that the image of a selectible fan (nonselectible) under a light confluent mapping is not selectible (nonselectible).

In the same paper the authors gave an example to show that nonselectibility is not preserved under light open mappings.

In this paper we are going to give a selectible fan and a light open mapping such that the image is a nonselectible fan. So selectibility between fans is not preserved under light open mappings, answering Question 1.1 and questions 1.2 and 1.3 for the light open mappings.

2. EXAMPLE

Example 2.1. There are a selectible fan X and an light open mapping g such that $g(X)$ is a nonselectible fan.

For each x and y in the 3-dimensional Euclidean space $\mathbb{R}^3$, denote by xy the convex arc joining x and y.

Now consider the following points in cylindrical coordinates in $\mathbb{R}^3$.

$$p = (0,0,0),\ a_0 = (\tfrac{1}{2}, \tfrac{3\pi}{4}, 0),\ a'_0 = (\tfrac{1}{2}, \tfrac{5\pi}{4}, 0),$$
$$a_n = (\tfrac{1}{2^{n-1}}, \tfrac{\pi}{2^n}, 0)$$
$$a_{0,m} = (\tfrac{1}{2}, \tfrac{3\pi}{4}, \tfrac{1}{m}),\ a'_{0,m} = (\tfrac{1}{2}, \tfrac{5\pi}{4}, \tfrac{1}{m}),$$
$$p_{0,m} = (\tfrac{1}{2^m}, \tfrac{5\pi}{8}, \tfrac{1}{m}),\ p'_{0,m} = (\tfrac{1}{2^m}, \pi, \tfrac{1}{m}),$$

$$a_{n,m} = (\tfrac{1}{2^{n-1}}(1+\tfrac{1}{m}), \tfrac{\pi}{2^n}, 0),\ p_{n,m} = (\tfrac{1}{m2^n}, \tfrac{3\pi}{2^{n+2}}, 0),$$

for every $n, m \in \mathbb{N}$.

Consider $T = \bigcup\{pa_n : n \in \mathbb{N}\} \cup pa_0 \cup pa'_0$. For each $m \in \mathbb{N}$, put $\widehat{pa'_m} = a'_{0,m}p'_{0,m} \cup p'_{0,m}a_{0,m} \cup a_{0,m}p_{0,m} \cup p_{0,m}a_{1,m} \cup \bigcup\{a_{n,m}p_{n,m} \cup p_{n,m}a_{n+1,m} : n \in \mathbb{N}\} \cup \{p\}$.

Note that the sequence $\{\widehat{pa'_m}\}$ converges to T. So

$$Y = T \cup (\bigcup\{\widehat{pa'_m} : m \in \mathbb{N}\})$$

is a countable fan with top p.

Note that the fan Y is homeomorphic to the plane fan Y' in Figure 1. Moreover, Y' is a rotation about 90° with respect to the fan Y.

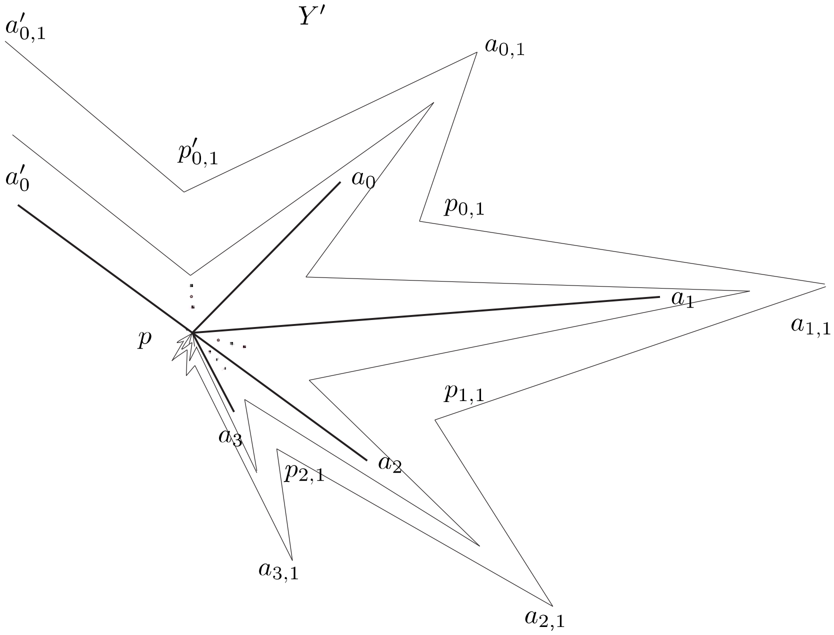

Figure 1: Fan Y'

Let $f : Y - (pa_0 \cup pa'_0) \to \mathbb{R}^3$ be defined by $f((r, \theta, z)) = (r, -\theta, -z)$. Put

$$X = Y \cup f(Y - (pa_0 \cup pa'_0)).$$

Then X is a countable plane fan with top p, which is homeomorphic to the plane fan X' in Figure 2.

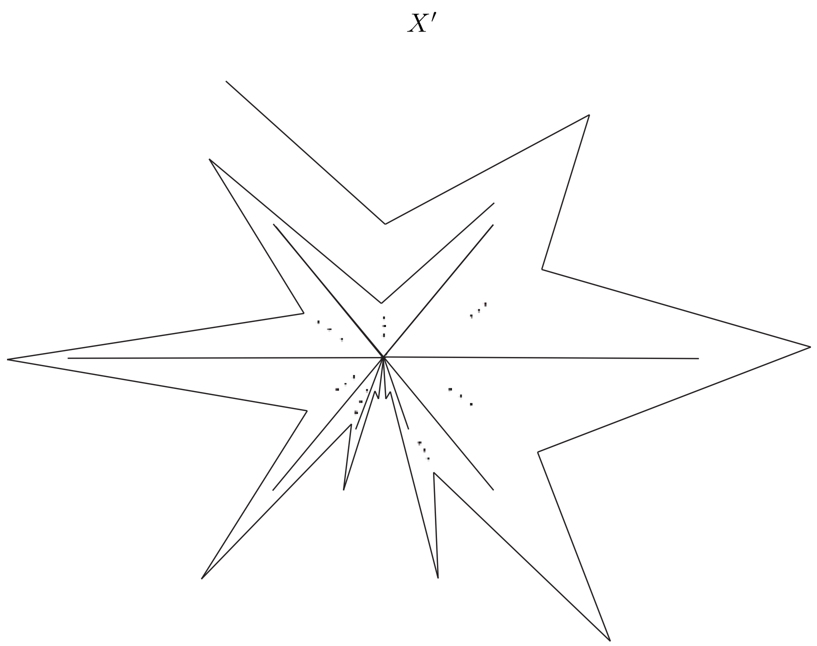

Figure 2: Fan X'

Claim: X is selectible.

In order to describe the selection of $C(X)$, let us consider the following set

$$\begin{aligned} F &= \bigcup\{pa_n : n \in \mathbb{N} \cup \{0\}\} \cup \bigcup\{pa'_n : n \in \mathbb{N} \cup \{0\}\} \\ &= T \cup f(T - (pa_0 \cup pa'_0)), \end{aligned}$$

where $a'_n = f(a_n)$ for each $n \in \mathbb{N}$.

For each $x \neq y \in F$ such that $p \in xy$, let $s(x, y)$ be the point in F such that $||s(x, y)|| = \frac{\left|||x|| - ||y||\right|}{2}$ in such a way that either $s(x, y) \in px$ if $||x|| > ||y||$ or $s(x, y) \in py$ if $||y|| > ||x||$; in other words, $s(x, y)$ is the middle point into the arc $xp \cup py$ ($|| * ||$ denotes the Euclidean norm of a point in $\mathbb{R}^3$ and $\big| * \big|$ denotes the absolute value of a real number).

Let $C(F,p)=\{K\in C(F) : p\in K\}$ and we define

$$\alpha : C(F,p)\times\mathbb{R}^2 \longrightarrow F$$

a mapping such that $\alpha(K,z)$ is the point of $pz\cap K$ having the greatest norm.

We define a selection $\sigma' : C(F) \longrightarrow F$ in the following way: Let $K\in C(F)$. If $p\notin K$, K is an arc $xy\subset F$. Without loss of generality, we can suppose that $||x||\geq||y||$, so it is natural to put $\sigma'(K)=x$.

If $p\in K$, put

$t_n=\alpha(K,a_n)$ and $t'_n=\alpha(K,a'_n)$ for each $n\in\mathbb{N}\cup\{0\}$,
$x_1=\alpha(K,4s(s(t_0,2t'_1),s(t'_0,2t_1)))$,
$x_n=2s(t_n,t'_n)$ for each $n>1$,
$y_0=p$,
$y_n=\alpha(K,2s(y_{n-1},3x_n))$ for each $n\in\mathbb{N}$.

So define

$$\sigma'(K)=\alpha(K,\lim y_n).$$

To show that σ' is a mapping. We will consider all possible cases for $\sigma'(K)$ for every $K\in C(F)$.

1. If $K=pt_n$ or $K=pt'_n$ for any $n\in\mathbb{N}\cup\{0\}$, then $\sigma'(K)=t_n$ or $\sigma'(K)=t'_n$.
   - (a) $K=pt_0$.
     Then $s(t_0,2t'_1)=\{\frac{t_0}{2}\}$ and $s(t'_0,2t_1)=p$, so $x_1=t_0$ and $x_n=p$ for each $n\in\mathbb{N}$; therefore, the sequence $\{y_n\}$ converges to t_0 and $\sigma'(K)=t_0$. Analogously, if $K=pt'_0$.
   - (b) $K=pt_1$.
     We have that $s(t_0,2t'_1)=p$ and $s(t'_0,2t_1)=t_1$, so $x_1=t_1$ and $x_n=p$ for each $n>1$. Thus, the sequence $\{y_n\}$ converges to t_1 and $\sigma'(K)=t_1$. Similarly, if $K=pt'_1$.
   - (c) $K=pt_n$ for any $n>1$.
     Then $x_j=p$ for every $j\neq n$ and $x_j=t_j$ if $j=n$, so $y_n=t_n$ for each $j>n$. Hence, the sequence $\{y_n\}$ converges to t_n and $\sigma'(K)=t_n$. Analogously, if $K=pt'_n$.
2. If $K=t_nt'_n$ for any $n\in\mathbb{N}\cup\{0\}$, then $\sigma'(K)\in pt_n$ if $||t_n||>||t'_n||$ or $\sigma'(K)\in pt'_n$ if $||t'_n||>||t_n||$.
   - (a) Suppose that $K=t_0t'_0$ and $||t_0||>||t'_0||$.
     We have that $s(t_0,2t'_1)=\{\frac{t_0}{2}\}$ and $s(t'_0,2t_1)=\frac{t'_0}{2}$. So $s(s(t_0,2t'_1),s(t'_0,2t_1))\in p\frac{t_0}{4}$, then $x_1\in pt_0$ and $x_n=p$ for

each $n \in \mathbb{N}$. Thus, the sequence $\{y_n\}$ converges to x_1 and $\sigma'(K) \in pt_0$.

(b) Suppose that $K = t_1t'_1$ and $||t_1|| > ||t'_1||$.
So $s(t_0, 2t'_1) = t'_1$, $s(t'_0, 2t_1) = t_1$, and $s(s(t_0, 2t'_1), s(t'_0, 2t_1)) \in p\frac{t_1}{2}$. Thus, $x_1 \in pt_1$ and $x_n = p$ for each $n \in \mathbb{N}$ and the sequence $\{y_n\}$ converges to x_1 and $\sigma'(K) \in pt_1$.

(c) $K = t_nt'_n$ and $||t_n|| > ||t'_n||$ for $n > 1$.
We have that $x_j = p$ for each $j \neq n$ and $x_j \in pt_j$ to $j = n$, so the sequence $\{y_m\}$ converges to x_n and $\sigma'(K) \in pt_n$.

(3) If $a'_0, a_0 \in K \subset pa'_0 \cup pa_0 \cup pa_1$, then $\sigma'(K) \in pa_0 \cup pa_1$.

(a) $||t_1|| \leq 1/4$.
We have that $s(t_0, 2t'_1) = \{\frac{a_0}{2}\}$ and $s(t'_0, 2t_1) \in p\frac{a'_0}{2}$. So $s(s(t_0, 2t'_1), s(t'_0, 2t_1)) \in p\frac{a_0}{4}$; in the other words, $x_1 \in pa_0$ and $x_n = p$ for each $n > 1$. Thus, the sequence $\{y_n\}$ converges to x_1. Hence, $\sigma'(K) \in pa_0$. Notice that $\sigma'(K)$ runs through the segment pa_0 in the sense from p to a_0, when $||t_1||$ goes from 0 to 1/4.

(b) $1/4 < ||t_1|| < 1/2$.
Then $s(t_0, 2t'_1) = \{\frac{a_0}{2}\}$ and $s(t'_0, 2t_1) \in p\frac{a'_0}{4}$. Since $||s(t_0, 2t'_1)|| \geq ||s(t'_0, 2t_1)||$, $s(s(t_0, 2t'_1), s(t'_0, 2t_1)) \in p\frac{a_0}{4}$; in other words, $x_1 \in pa_0$ and $x_n = p$ for each $n > 1$. So the sequence $\{y_n\}$ converges to x_1 and $\sigma'(K) \in pa_0$. In this case, $\sigma'(K)$ runs through the segment pa_0 in the sense from a_0 to p, when $||t_1||$ goes from 1/4 to 1/2.

(c) $1/2 < ||t_1|| < 1$.
So $s(t_0, 2t'_1) = \{\frac{a_0}{2}\}$ and $s(t'_0, 2t_1) \in \frac{a_1}{4}\frac{3a_1}{4}$. Note that $||s(t_0, 2t'_1)|| \geq ||s(t'_0, 2t_1)||$. Thus, $s(s(t_0, 2t'_1), s(t'_0, 2t_1)) \in p\frac{a_1}{4}$; in other words, $x_1 \in pa_1$ and $x_n = p$ for each $n > 1$. Hence, the sequence $\{y_n\}$ converges to x_1 and $\sigma'(K) \in pa_1$. Here, $\sigma'(K)$ runs through the segment pa_1 in the sense from p to a_1, when $||t_1||$ goes from 1/2 to 1.

(4) If $a_1 \in K \subset pa'_0 \cup pa_0 \cup pa_1$, then $\sigma'(K) = a_1$.

In this case, we have that $s(t_0, 2t'_1) \in p\{\frac{a_0}{2}\}$ and $s(t'_0, 2t_1) \in \frac{3a_1}{4}a_1$. So $s(s(t_0, 2t'_1), s(t'_0, 2t_1)) \in \frac{a_1}{4}\frac{a_1}{2}$; in other words, $x_1 = a_1$ and $x_n = p$ for each $n > 1$. Thus, the sequence $\{y_n\}$ converges to x_1 and $\sigma'(K) = a_1$.

(5) If $K = F$, then $\sigma'(F) = p$.

Since $K = F$, $t_n = a_n$, $t'_n = a'_n$, and $||t_n|| = ||t'_n||$ for all $n \in \mathbb{N} \cup \{0\}$. So $x_n = p$ for $n \in \mathbb{N}$, the sequence $\{y_n\}$ converges to p, and $\sigma'(F) = p$.

Using conditions (1)–(5), we can see that σ' is a continuous selection for $C(F)$.

Now let $\beta : X \longrightarrow F$ be a retraction from X onto F such that for each $(n,m) \in (\mathbb{N} \cup \{0\}) \times \mathbb{N}$, $\beta(a_{n,m}) = a_n$, $\beta(f(a_{n,m})) = a'_n$, and $\beta(p_{n,m}) = p = \beta(f(p_{n,m}))$.

We define a partial order $\leq_p$ on X with respect to the point p as follows: Let $x, y \in X$, $x \leq_p y$ if and only if $px \subset py$.

Notice that if $K \in C(X)$ such that $K \cap F = \emptyset$, then K is an arc such that $K \subset \widehat{pa'_m}$ or $K \subset f(\widehat{pa'_m})$ for some $m \in \mathbb{N}$, and the set

$$\beta^{-1}(\sigma'(\beta(K))) \cap K = (\beta|_K)^{-1}(\sigma'(\beta(K)))$$

is finite.

We define a function $\sigma : C(X) \longrightarrow X$ by

$$\sigma(K) = \begin{cases} \sigma'(K \cap F), & \text{if } K \cap F \neq \emptyset, \\ \min_{\leq_p}(\beta|_K)^{-1}(\sigma'(\beta(K))), & \text{if } K \cap F = \emptyset. \end{cases}$$

One can verify that σ is a selection for $C(X)$, and so X is a selectible fan.

Now we define an equivalence relation in X: Let $x, y \in X$ and let $x \sim y$ if and only if either $y = f(x)$ or $x \in pa_0$, $y \in pa'_0$, and $||x|| = ||y||$.

Consider $Z = X/\sim$. Clearly, Z is homeomorphic to the fan in Figure 3.

Let g be the quotient mapping from X to Z. Notice that if $\widehat{z} \in Z$, $g^{-1}(\widehat{z}) = \{z, f(z)\}$, when $\widehat{z} \neq \widehat{p}$, and $g^{-1}(\widehat{z}) = \{p\}$, if $\widehat{z} = \widehat{p}$. So g is a light mapping. In order to see that g is open we will use the following theorem ([1, Theorem. 2.4]).

Theorem 2.2. *Let $f : X \longrightarrow Y$ be a map between continua. Then f is open if and only if for each sequence $\{y_n\}_{n\in\mathbb{N}}$ in Y such that $\lim_{n\to\infty} y_n = y$, for some point $y \in Y$, and for any $x \in f^{-1}(y)$ there exists a sequence $\{x_n\}_{n\in\mathbb{N}}$ in X such that $\lim_{n\to\infty} x_n = x$ and $x_n \in f^{-1}(y_n)$, for each $n \in \mathbb{N}$.*

Let $\{\widehat{z}_n\}_{n\in\mathbb{N}}$ be a sequence of Z such that $\lim_{n\to\infty} \widehat{z}_n = \widehat{z}$ for some $\widehat{z} \in Z$. Since $g^{-1}(\widehat{z}) = \{z, f(z)\}$ and $g^{-1}(\widehat{z}_n) = \{z_n, f(z_n)\}$ for each $n \in \mathbb{N}$, by the construction of Z, the sequence $\{z_n\}_{n\in\mathbb{N}}$ converges to z and by continuity of f, $\{f(z_n)\}_{n\in\mathbb{N}}$ converges to $f(z)$. Then, by Theorem 2.2, g is open.

Notice that Z is of type N between the point $\widehat{p}$ and the point $\widehat{a}_0 = \{a_0, a'_0\}$. So Z does not have the bend intersection property. Thus, by [5, Corollary, p. 548], Z is a non-selectible fan.

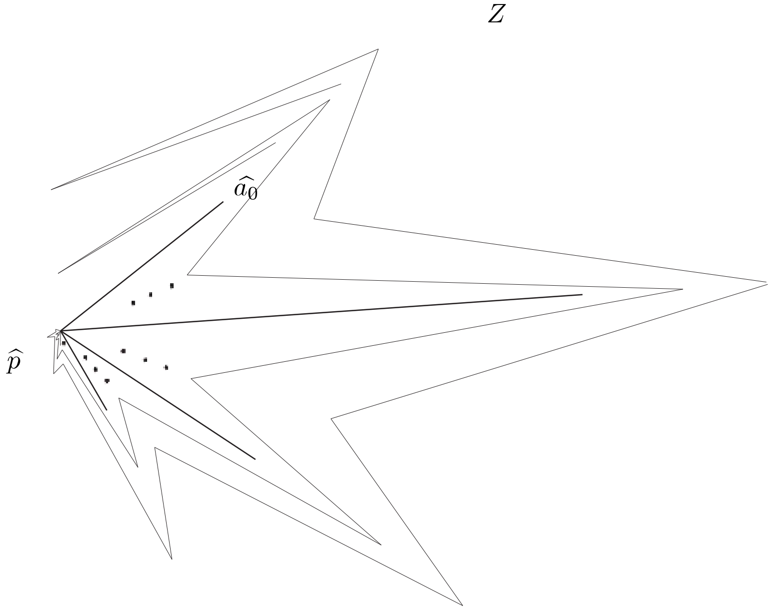

Figure 3: Fan Z

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Capítulo 3

Artículo: General properties on Pseudo-contractibility

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General Properties on Pseudo-contractibility

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Abstract

In this paper we are going to give general facts about pseudo-homotopies and pseudo-contractibility. As a consequence of these, we find several conditions that obstruct pseudo-contractibility and we present examples of pseudo-contractible and non pseudo-contractible continua.

Keywords: Pseudo-contractible, contractible, Property b), unicoherent, acyclic, homotopy equivalent, trivial shape, curves, dendroids.
2000 MSC: Primary, 54C05, 54C15, 54C55; Secondary, 54B17

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1. Introduction

R. H. Bing introduced the notion of pseudo-contractibility. However W. Kuperberg was the first to prove that the notions of pseudo-contractibility and contractibility are different (see [12]); he asked whether or not the sin $\frac{1}{x}$ curve is pseudo-contractible. H. Katsuura proves in [8] that the sin $\frac{1}{x}$ curve is not pseudo-contractible with factor space itself. In the same paper H. Katsuura proves that if Y is a nondegenerate indecomposable continuum such that each one of their composants is arcwise connected and X is a continuum having a proper nondegenerate arc component, then X is not pseudo-contractible with factor Y. Other questions related with this topic are the following:
Question 1.([8, Question 1, pp. 1136]) Is the sin $\frac{1}{x}$ curve pseudo-contractible relative to the pseudoarc?

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Question 2.([12, Problem 118]) Is the pseudoarc pseudo-contractible with factor pseudoarc?

W. Dębski proves in [5] that the sin $\frac{1}{x}$ curve is not pseudo-contractible. On the other hand, M. Sobolewsky [18] shows that the only chainable continuum that is pseudo-contractible is the arc, particularly the pseudoarc is another example of a continuum that is not pseudo-contractible. The reader is referred to [1], [7], [8], [12] and [18] for more information about the results above.

In this work we are going to give general facts and results about pseudo-homotopies and pseudo-contractibility. As a consequence of this study we find several conditions that obstruct pseudo-contractibility and we present more examples of pseudo-contractible and non pseudo-contractible continua.

2. Preliminaries

A *continuum* means a nonempty compact connected metric space. A *curve* is a one dimensional continuum. A topological space X is said to be *continuumwise connected* provided that every pair of points of X are contained in a subcontinuum of X. A *mapping* means a continuous function. If there exists a homeomorphism $f : X \to Y$, we say that X is homeomorphic to Y and we write $X \approx Y$. A continuum X is said to be *unicoherent* provided that for each pair of subcontinua H and K of X such that $X = H \cup K$, $H \cap K$ is connected, and it is *hereditarily unicoherent* if each subcontinuum of X is unicoherent. An *arc* is understood as a homeomorphic image of the closed unit interval $I = [0, 1]$ of the real line. If every two points of a space Z can be joined by an arc lying in Z, then Z is called *arcwise connected.*

A continuum is said to be a *dendroid*, if it is arcwise connected and hereditarily unicoherent. A continuum is said to be *decomposable* provided that it can be written as the union of two proper subcontinua and it is called *hereditarily decomposable* if each nondegenerate subcontinuum of X is decomposable. A λ-*dendroid* means a hereditarily unicoherent and hereditarily decomposable continuum. It is well known that dendroids are λ-dendroids ([15, p. 226]).

Let $C(X, Y)$ be the topological space of all mappings from X to Y topologized with the compact-open topology. It is well known that if Y is a compact metric space, then the compact-open topology coincides with the topology obtained with the supreme metric on $C(X, Y)$.

Let $\{X_\alpha\}_{\alpha\in I}$ be a family of topological spaces. The product space (X,τ) is defined by $X=\prod_{\alpha\in I}X_\alpha=\{f:I\to\bigcup_{\alpha\in I}X_\alpha : f(x_\alpha)\in X_\alpha,\ \text{for all } \alpha\in I\}$ with τ the product topology.
Notation: $f(\alpha)=x_\alpha$. We use $(x_\alpha)_{\alpha\in I}$ instead of $f:I\to\bigcup_{\alpha\in I}X_\alpha\in\prod_{\alpha\in I}X_\alpha$.

Let (X_1,τ_1) and (X_2,τ_2) be topological spaces such that $X_1\cap X_2=\emptyset$. The *free union* of X_1 and X_2 is the topological space (X,τ), where $X=X_1\cup X_2$ and $U\in\tau$ if and only if $U\cap X_i\in\tau_i$ for each $i=1,2$. The free union of X_1 and X_2 is denoted by X_1+X_2.

Let (X_1,τ_1) and (X_2,τ_2) be topological spaces such that $X_1\cap X_2=\emptyset$, let A be a nonempty closed subset of X_1 and let $f:A\to X_2$ be a mapping. Let D be the partition of X_1+X_2 given by
$D=\{\{p\}\cup f^{-1}(p):p\in f(A)\}\cup\{\{x\}:x\in X_1+X_2\setminus(A\cup f(A))\}$.
The decomposition space thus obtained is denoted by $X_1\cup_f X_2$ and it is called the *attached space*.

If X and Y are disjoint continua, then the attached space $X\cup_f Y$ is a continuum ([15, Theorem 3.20]).

3. Pseudo-homotopy

Definition 1. *Let X and Y be topological spaces and let $f,\ g:X\to Y$ be mappings. We say that f is homotopic to g (or f and g are homotopic), written by $f\simeq g$, if there exists a mapping (called homotopy) $H:X\times I\to Y$ satisfying $H(x,0)=f(x)$ y $H(x,1)=g(x)$ for each $x\in X$.*

Definition 2. *Let X and Y be topological spaces and let $f,\ g:X\to Y$ be mappings. We say that f is pseudo-homotopic to g (or f and g are pseudo-homotopic) if there are a continuum C, points $a,\ b\in C$ and a mapping $H:X\times C\to Y$ such that $H(x,a)=f(x)$ and $H(x,b)=g(x)$ for each $x\in X$. We write $f\simeq_C g$ to say that f is pseudo-homotopic to g with factor space C. The mapping H is called a pseudo-homotopy between f and g with factor space C.*

Remark 3. *Note that if $f\simeq_C g$ and K is a subcontinuum of C such that $a,b\in K$ then $f\simeq_K g$. In particular, if $I_{ab}\subset C$ is an irreducible continuum between a and b then $f\simeq_{I_{ab}} g$. Moreover if I_{ab} is an arc from a to b, then $f\simeq g$. In particular $f\simeq_C g$ implies $f\simeq g$ if C is arcwise connected.*

Remark 4. *Let $f,\ g:X\to Y$ be mappings. If f is pseudo-homotopic to g and Z is a subset of X, then $f|_Z$ is pseudo-homotopic to $g|_Z$.*

Theorem 5. *Let* $f,\ g : X \to Y$ *be mappings between topological spaces. If* $f \simeq_C g$ *and there exist a continuum* K *and an onto mapping* $h : K \to C$*, then* $f \simeq_K g$*. Moreover by Remark* 3*, if there are a continuum* K' *and an onto mapping from* K' *to some subcontinuum* $C' \subset C$ *such that* $a,\ b \in C'$*, then* $f \simeq_{K'} g$*.*

Proof. Let H be a mapping and let C and a, b as in Definition 2. Let $a' \in h^{-1}(a)$, $b' \in h^{-1}(b)$ and let $G : X \times K \to X \times C$ be a function given by $G(x,k) = (x, h(k))$.
The mapping $H' : X \times K \to Y$ given by $H'(x,k) = (H \circ G)(x,k)$ is a pseudo-homotopy between f and g. □

Definition 6. *Let* X *and* Y *be continua. The continua* X *and* Y *are said to be continuously equivalent provided that there are two onto mappings* $f : X \to Y$ *and* $g : Y \to X$*.*

Corollary 7. *Let* $f,\ g : X \to Y$ *be mappings and let* C *and* D *be continuously equivalent continua. Then* $f \simeq_C g$ *if and only if* $f \simeq_D g$*.*

Proof. The proof follows from Theorem 5. □

Corollary 8. *Let* C_1 *and* C_2 *be continua such that* $C_1 \approx C_2$ *and let* $f,\ f' : X \to Y$ *be mappings. Then* $f \simeq_{C_1} f'$ *if and only if* $f \simeq_{C_2} f'$*.*

Let f, g in $C(X,Y)$. We say that f is related to g, written $f \simeq_* g$, if and only if there is a continuum K, such that $f \simeq_K g$

Theorem 9. *The relation* $\simeq_*$ *is an equivalence relation in* $C(X,Y)$*.*

Proof. The reflexive and symmetric properties are trivial.
We only need to prove the transitivity. Let $f,\ g,\ h : X \to Y$ be mappings, such that $f \simeq_* g$ and $g \simeq_* h$, then there exist, continua C_1, C_2, points $a_1,\ b_1 \in C_1$, points $a_2,\ b_2 \in C_2$ and mappings $H_1 : X \times C_1 \to Y$ and $H_2 : X \times C_2 \to Y$ such that $H_1(x, a_1) = f(x)$, $H_1(x, b_1) = g(x)$ and $H_2(x, a_2) = g(x)$, $H_2(x, b_2) = h(x)$ for each $x \in X$.
Without loss of generality, we assume that $C_1 \cap C_2 = \emptyset$. We consider $j : \{b_1\} \to C_2$ given by $j(b_1) = a_2$ and $D = C_1 \cup_j C_2$. Then we define a function $H : X \times D \to Y$ by

$$H(x,d) = \begin{cases} H_1(x,d) & if \quad d \in C_1 \\ H_2(x,d) & if \quad d \in C_2. \end{cases}$$

It is clair that H is a pseudo-homotopy between f and h. □

The equivalence classes in $C(X,Y)$ under the relation $\simeq_*$ are called pseudo-homotopy classes.

Theorem 10. *Let X and Y be compact metric spaces and let $f,\ g : X \to Y$ be mappings. The mappings f and g are pseudo-homotopic if and only if there exist a continuum in $C(X,Y)$ containing f and g.*

Proof. Suppose $f \simeq_C g$. For every $c \in C$ we define the mapping $h_c : X \to Y$ given by $h_c(x) = H(x,c)$, where H is the pseudo-homotopy between f and g. Then the function $G : C \to C(X,Y)$ defined by $G(c) = h_c$ is continuous. Since $C(X,Y)$ is a Hausdorff space and $G(C) \subset C(X,Y)$ then $G(C)$ is a Hausdorff space, thus $G(C)$ is metrizable ([11, §41, VI, Theorem 3]). So, $G(C)$ is a continuum containing f and g.
Conversely, let $f,\ g \in C(X,Y)$ and let $H \subset C(X,Y)$ be a continuum containing f and g. The function $F : X \times H \to Y$ given by $F(x,h) = h(x)$ is continuous and it satisfies $F(x,f) = f(x)$ and $F(x,g) = g(x)$ for all $x \in X$. ☐

Corollary 11. *Let X and Y compact metric spaces. Then every pseudo-homotopy class is continuumwise connected.*

Corollary 12. *Let X, Y be compact metric spaces. Every pair of mappings $f,\ g : X \to Y$ are pseudo-homotopic if and only if the space $C(X,Y)$ is continuumwise connected.*

Regarding the composition of functions we have the following results.

Theorem 13. *Let $h : Y \to Z,\ k : W \to X$ and $f,\ g : X \to Y$ be mappings, if $f \simeq_C g$ then $h \circ f \simeq_C h \circ g$ and $f \circ k \simeq_C g \circ k$.*

Proof. Since $f \simeq_C g$, there exist points $a,\ b \in C$ and a mapping $H : X \times C \to Y$ such that $H(x,a) = f(x)$ and $H(x,b) = g(x)$ for each $x \in X$.
To prove the first part we consider the function $G : X \times C \to Z$ defined by $G(x,c) = (h \circ H)(x,c)$. The function G is a pseudo-homotopy between $h \circ f$ and $h \circ g$.
On the other hand, the mapping $F : W \times C \to Y$ given by $H(z,c) = F(k(z),c)$ is a pseudo-homotopy between $f \circ k$ and $g \circ k$. ☐

Theorem 14. *Let $f,\ f' : X \to Y$ and $g,\ g' : Y \to Z$ be mappings such that $f \simeq_{C_1} f'$ and $g \simeq_{C_2} g'$, then $g \circ f$ is pseudo-homotopic to $g' \circ f'$.*

Proof. Since $f \simeq_{C_1} f'$ and $g \simeq_{C_2} g'$, there are points $a_1,\ b_1 \in C_1,\ a_2,\ b_2 \in C_2$ and mappings $H_1 : X \times C_1 \to Y$ and $H_2 : Y \times C_2 \to Z$ such that $H_1(x, a_1) = f(x)$, $H_1(x, b_1) = f'(x)$ for each $x \in X$ and $H_2(y, a_2) = g(y)$, $H_2(y, b_2) = g'(y)$ for each $y \in Y$.
We consider the continuum $C = C_1 \times C_2$ and the points $\hat{a_0} = (a_1, a_2)$, $\hat{b_0} = (b_1, b_2) \in C_1 \times C_2$, then the function $F : X \times (C_1 \times C_2) \to Z$ defined by $F(x, (c_1, c_2)) = H_2(H_1(x, c_1), c_2)$ is a pseudo-homotopy between $g \circ f$ and $g' \circ f'$. □

Let $\{X_j\}_{j\in J}$ be a family of topological spaces and let $\prod_{j\in J} X_j$ be the product space. It is well known that the projection functions $\pi_i : \prod_{j\in J} X_j \to X_i$ defined by $\pi_i((x_j)_{j\in J}) = x_i$, for all $i \in J$, are continuous. Theorem 15 below follows from Theorem 13.

Theorem 15. *Let $\{Y_\alpha\}_{\alpha\in I}$ be a family of topological spaces such that $\prod_{\alpha\in I} Y_\alpha$ is a compact metric space. Let $f,\ g : X \to \prod_{\alpha\in I} Y_\alpha$ be mappings. If f is pseudo-homotopic to g, then so are $\pi_\alpha \circ f$ and $\pi_\alpha \circ g$.*

Corollary 16. *Let $\{Y_n\}_{n\in\mathbb{N}}$ be a family of compact metric spaces. Let $f,\ g : X \to \prod_{n\in\mathbb{N}} Y_n$ be mappings. Then f and g are pseudo-homotopic if and only if the mappings $\pi_n \circ f$ and $\pi_n \circ g$ are pseudo-homotopic.*

Proof. Let $f,\ g : X \to \prod_{n\in\mathbb{N}} Y_n$ be mappings. Suppose that $\pi_n \circ f$ and $\pi_n \circ g$ are pseudo-homotopic for each $n \in \mathbb{N}$. Then for every $n \in \mathbb{N}$, there exist a continuum C_n, points $a_n,\ b_n \in C_n$ and mappings $H_n : X \times C_n \to Y_n$ such that $H_n(x, a_n) = (\pi_n \circ f)(x)$ and $H_n(x, b_n) = (\pi_n \circ g)(x)$.
Notice that $C = \prod_{n\in\mathbb{N}} C_n$ is a continuum. Let $a = (a_n)_{n\in\mathbb{N}}$, $b = (b_n)_{n\in\mathbb{N}} \in C$. The function $H : X \times C \to \prod_{n\in\mathbb{N}} Y_n$ defined by $H(x, c) = (H_n(x, c_n))_{n\in\mathbb{N}}$ is a pseudo-homotopy between f and g, where $c = (c_n)_{n\in\mathbb{N}}$.
The converse follows of Theorem 15. □

4. Pseudo-contractibility

Definition 17. *A topological space X is said to be contractible if its identity mapping is homotopic to a constant mapping x_0 in X, i.e., if there exists a mapping $H : X \times [0, 1] \to X$ satisfying $H(x, 0) = x$ and $H(x, 1) = x_0$, for each $x \in X$.*

Definition 18. *A topological space X is said to be pseudo-contractible if its identity mapping is pseudo-homotopic to a constant mapping in X, i.e.,* *if there exist a continuum C, points $a,\ b \in C$, $x_0 \in X$ and a mapping $H : X \times C \to X$ satisfying $H(x,a) = x$ and $H(x,b) = x_0$ for each $x \in X$.*

Notice that X is pseudo-contractible if and only if each mapping $f : X \to X$ is pseudo-homotopic (homotopic) to a constant mapping.

The following example was given by W. Kuperberg and is the first example showing that the concepts of contractibility and pseudo-contractibility are different. We describe and draw here this example for the interested readers (see Figure 1).

Example 19. *(W. Kuperberg)*

Let $\mathbb{C}$ be the complex plane and let $X_0 = \{\frac{t+2}{t+1}e^{it} : t \in [0,\infty)\}$ be the spiral approaching the unit circle S^1. Let $X = X_0 \cup \{x : |x| \leq 1\} \subset \mathbb{C}$. X is not contractible because it is not arcwise connected.
Consider $C = X_0 \cup \{x : |x| = 1\} \cup X_1 \subset \mathbb{C}$, where $X_1 = \{x \in \mathbb{C} : Im(x) = 0,\ 0 \leq Re(x) \leq 1\}$.
We define a mapping $H : X \times C \to X$ as follow:

1. $H(\frac{t+2}{t+1}e^{it}, \frac{t'+2}{t'+1}e^{it'}) = \frac{t+t'+2}{t+t'+1}e^{i(t+t')}$ *if* $t,\ t' \in [0,\infty)$.
2. $H(x, \frac{t+2}{t+1}e^{it}) = xe^{it}$ *if* $|x| \leq 1,\ t \in [0,\infty)$.
3. $H(x,x') = xx'$ *if* $|x| \leq 1,\ |x'| = 1$ *or* $x' \in X_1$
4. $H(\frac{t+2}{t+1}e^{it}, x) = xe^{it}$ *if* $t \in [0,\infty),\ |x| = 1$ *or* $x \in X_1$

We have that $H(x,2) = x$ and $H(x,0) = 0$ for each $x \in X$, then X is pseudo-contractible.

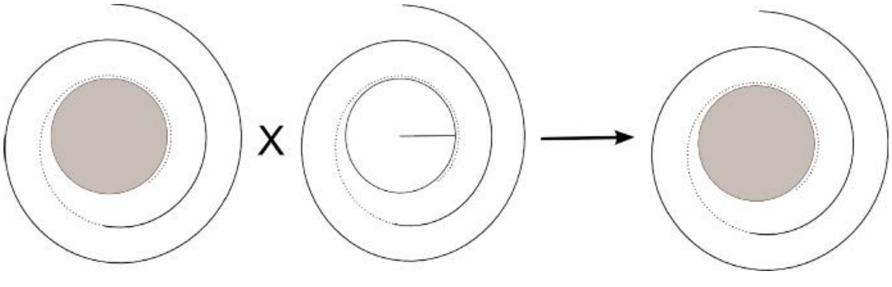

Figure 1: Pseudo-contractible continuum.

The following example shows that a continuum can be pseudo-contractible with more than one factor space.

Example 20. *The Comb space is pseudo-contractible with factor space itself. Consider* $P = ((\{1/n\} : n \in \mathbb{N} \cup \{0\}) \times [0,1]) \cup ([0,1] \times \{0\})$.
We consider the mapping $H : P \times P \to P$ *defined by* $H((x_1, y_1), (x_2, y_2)) = (x_1x_2, y_1y_2)$. H *satisfies that* $H((x, y), (1, 1)) = (x, y)$ *and* $H((x, y), (0, 0)) = (0, 0)$ *for each* $(x, y) \in P$. *Thus* H *is a pseudo-homotopy between the identity mapping and a constant mapping.*

The following result is a consequence of Remark 3.

Theorem 21. *If a continuum* X *is pseudo-contractible with (locally connected) arcwise-connected factor space* C*, then* X *is contractible.*

In general, from Theorem 5 and Theorem 7 we have the following four results.

Corollary 22. *If* X *is pseudo-contractible with factor space* C *and there exists an onto mapping* $f : C' \to C$*, then* X *is pseudo-contractible with factor space* C'.

Proof. If $id_X \simeq_C x_0$ and $f : C' \to C$ is an onto mapping, then Theorem 5 implies that $id_X \simeq_{C'} x_0$. □

Corollary 23. *Let* C_1 *and* C_2 *be continua such that* C_1 *is continuously equivalent to* C_2*. Then* X *is pseudo-contractible with factor space* C_1 *if and only if it is pseudo-contractible with factor space* C_2.

Corollary 24. *If a topological space* X *is contractible, then* X *is pseudo-contractible with any factor continuum.*

Proof. The proof is a consequence of Urysohn's Lemma and Theorem 5. □

Recall that by Hahn-Mazurkiewicz's Theorem every locally connected continuum C is the continuous image to the interval and by Urysohn's Lemma there exist continuous functions from every normal space to the interval. So each locally connected continuum C is continuously equivalent to the interval.

Theorem 25. *Let* X *be a topological space, the following are equivalent:*

1. X *is pseudo-contractible for any factor continuum.*
2. X *is pseudo-contractible for any factor locally connected continuum* C.

3. *X is pseudo-contractible for some factor locally connected continuum C.*

4. *X is pseudo-contractible for some arcwise-connected factor space.*

5. *X is pseudo-contractible for any arcwise-connected factor space.*

6. *X is pseudo-contractible for some factor space C such that a and b can be joined with an arc in C, where C, a and b are as in Definition 18.*

7. *X is contractible.*

Remark 26. *If X is pseudo-contractible non arcwise-connected continuum, then by Theorem* 21*, the factor space must be non arcwise-connected. Therefore the example given by W. Kuperberg cannot be pseudo-contractible with factor space an arcwise-connected continuum because it is not contractible.*

Definition 27. *Let X be a topological space and let A be a closed subset of X. A retraction from X onto A is a mapping $r : X \to A$ such that $r(a) = a$ for each $a \in A$. The set A is called a retract of X .*

We will see that pseudo-contractibility (as well as contractibility) is preserved under retractions.

Theorem 28. *Let X be a pseudo-contractible space. If A is a retract of X, then A is pseudo-contractible.*

Proof. Suppose X is pseudo-contractible. Then there exist a continuum C, points $a,\ b \in C$, $x_0 \in X$ and a mapping $H : X \times C \to X$ satisfying $H(x, a) = x$ and $H(x, b) = x_0$ for each $x \in X$.
Since A is a retract of X, there exists a mapping $r : X \to A$ such that $r(y) = y$ for each $y \in A$. Let $a_0 = r(x_0) \in A$
Consider the mapping $i : A \times C \to X \times C$ given by $i(y, c) = (y, c)$.
To show that A is pseudo-contractible consider the mapping $G : A \times C \to A$ defined by $G(y, c) = (r \circ H \circ i)(y, c)$. The function G is a pseudo-homotopy between the identity and the constant mapping a_0. □

Remark 29. *Is not difficult to see that if $X \approx Y$ and X is pseudo-contractible then so does Y.*

Theorem 30. *Let X and Y be topological spaces. The spaces X and Y are pseudo-contractible if and only if the product space $X \times Y$ is pseudo-contractible.*

Proof. If X and Y are pseudo-contractible, there exist continua C_1, C_2, points a_1, $b_1 \in C_1$, a_2, $b_2 \in C_2$, $x_0 \in X$, $y_0 \in Y$ and mappings $H_1 : X \times C_1 \to X$, $H_2 : Y \times C_2 \to Y$ satisfying $H_1(x, a_1) = x$, $H_1(x, b_1) = x_0$ y $H_2(y, a_2) = y$, $H_2(y, b_2) = y_0$ for each $x \in X$ and each $y \in Y$.
Consider the continuum $C_1 \times C_2$ and the points (a_1, a_2), $(b_1, b_2) \in C_1 \times C_2$. The function $H : (X \times Y) \times (C_1 \times C_2) \to X \times Y$ defined by $H((x, y), (c_1, c_2)) = (H_1(x, c_1), H_2(y, c_2))$ is a pseudo-homotopy between the identity and the constant mapping (x_0, y_0).
To prove the other way, let $(x_0, y_0) \in X \times Y$. We can see that $X \times \{y_0\} \approx X$ and $\{x_0\} \times Y \approx Y$. Since $X \times Y$ is pseudo-contractible, Theorem 28 and Remark 29 imply that X and Y are pseudo-contractibles. □

Corollary 31. *Let $\{X_n\}_{n\in\mathbb{N}}$ be a sequence of topological spaces. The space X_n is pseudo-contractible for all $n \in \mathbb{N}$ if and only if the product space $\prod_{n\in\mathbb{N}} X_n$ is pseudo-contractible.*

Proof. If $\{X_n\}_{n\in\mathbb{N}}$ is a sequence of pseudo-contractible topological spaces, then there exist $\{C_n\}_{n\in\mathbb{N}}$ a sequence of continua, points a_n, $b_n \in C_n$, $x_n^0 \in X_n$ and mappings $H_n : X_n \times C_n \to X_n$, satisfying $H_n(x, a_n) = x$, $H_n(x, b_n) = x_n^0$ for each $x \in X_n$ and each $n \in \mathbb{N}$.
Consider the continuum $C = \prod_{n\in\mathbb{N}} C_n$ and the points $(a_n)_{n\in\mathbb{N}}$, $(b_n)_{n\in\mathbb{N}} \in C$. We define the function $H : (\prod_{n\in\mathbb{N}} X_n) \times C \to \prod_{n\in\mathbb{N}} X_n$ by $H((x_n)_{n\in\mathbb{N}}, (c_n)_{n\in\mathbb{N}}) = (H_n(x_n, c_n))_{n\in\mathbb{N}}$. The mapping H is a pseudo-homotopy between the identity and the constant mapping $(x_n^0)_{n\in\mathbb{N}}$.
To prove the converse, let $(x_n^0)_{n\in\mathbb{N}} \in \prod_{n\in\mathbb{N}} X_n$, we can see that $X_n \times \prod_{j\neq n}\{x_j^0\} \approx X_n$ for each $n \in \mathbb{N}$. Given that $\prod_{n\in\mathbb{N}} X_n$ is pseudo-contractible, it follows from Theorem 28 and Remark 29 that X_n is pseudo-contractible for each $n \in \mathbb{N}$. □

Corollary 32. *Let $\{X_\alpha\}_{\alpha\in I}$ be a family of topological spaces. If $\prod_{\alpha\in I} X_\alpha$ is pseudo-contractible, then X_α is pseudo-contractible for each $\alpha \in I$.*

Proof. The proof is a consequence of Theorem 28 and Remark 29. □

Corollary 33. *Let X be a topological spaces. The following five statements are equivalent:*

1. *X is pseudo-contractible.*

2. *X^n is pseudo-contractible for each $n \in N$.*

3. X^n *is pseudo-contractible for some* $n \in N$.

4. The cylinder $X \times [0,1]$ *is pseudo-contractible.*

5. $\prod_{n\in\mathbb{N}} X_n$ *is pseudo-contractible, where* $X_n = X$ *for each* $n \in \mathbb{N}$.

5. Pseudo-contractibility with respect to Y

Definition 34. *Let* X *and* Y *be a topological spaces. We say that* X *is pseudo-contractible (contractible) with respect to* Y *if each mapping* $f : X \to Y$ *is pseudo-homotopic (homotopic) to a constant mapping.*

Definition 35. *A subspace* Z *of* X *is said to be pseudo-contractible (contractible) in* X *if the inclusion mapping is pseudo-homotopic (homotopic) to a constant mapping in* X.

Note that if $Z \subset X$ and Z is pseudo-contractible (contractible) with respect to X, then Z is pseudo-contractible (contractible) in X

Theorem 36. *If* X *is pseudo-contractible with respect to* Y *and* Y *is continuumwise connected, then every pair of mappings* $f,\ g : X \to Y$ *are pseudo-homotopic. In particular if* X *is pseudo-contractible with respect to* Y *and* Y *is a continuum, every pair of mappings* $f,\ g : X \to Y$ *are pseudo-homotopic.*

Proof. Let $f,\ g : X \to Y$ be mappings. Since X is pseudo-contractible with respect to Y, $f \simeq_{C_1} y_1$ and $g \simeq_{C_2} y_2$, i.e., there exist $H_1 : X \times C_1 \to Y$ and $H_2 : X \times C_2 \to Y$ mappings, points $a_1,\ b_1 \in C_1$ and $a_2,\ b_2 \in C_2$ such that $H_1(x, a_1) = f(x)$, $H_1(x, b_1) = y_1$ and $H_2(x, a_2) = y_2$, $H_2(x, b_2) = g(x)$.
Since Y is continuumwise connected, there exists a continuum K joining y_1 with y_2. Thus we consider the attached continuum $C = C_1 \cup_j K \cup_l C_2$, where $j : \{b_1\} \to K$, $l : \{y_2\} \to C_2$ are mappings defined by $j(b_2) = y_1$ and $l(y_2) = a_2$.
The mapping $F : X \times C \to Y$ given by

$$F(x,c) = \begin{cases} H_1(x,c) & if \quad c \in C_1 - \{b_1\} \\ y_1 & if \quad c = \{b_1, y_1\} \\ c & if \quad c \in K - \{y_1, y_2\} \\ y_2 & if \quad c = \{a_2, y_2\} \\ H_2(x,c) & if \quad c \in C_2 - \{a_2\} \end{cases}$$

is a pseudo-homotopy between f and g. □

Note that whether X is a topological space and Y is continuumwise connected, then every pair of constant mappings from X into Y are pseudo-homotopic with factor space $Y' \subset Y$, where Y' is the subcontinuum containing the image of both constant mappings. The projection mapping of the product $X \times Y'$ over Y' works in this case. On the other hand if Z is a continuumwise connected pseudo-contractible space, then Z is pseudo-contractible to any constant mapping. In particular we have those results when Y and Z are continua.

Corollary 37. *Let X be a compact metric space and let Y be a continuumwise connected (continuum). X is pseudo-contractible with respect to Y if and only if the space $C(X, Y)$ is continuumwise connected.*

Proof. Let $f,\ g : X \to Y$ be mappings. Since X is pseudo-contractible with respect to Y, then by Theorem 36, $f \simeq_C g$ and by Theorem 10, there exists a continuum joining f with g in $C(X, Y)$, thus $C(X, Y)$ is continuumwise connected.
Inversely, let $f,\ g : X \to Y$ be mappings where g is a constant mapping. By hypothesis there exists a continuum K in $C(X, Y)$ joining f with g. By Theorem 10, X is pseudo-contractible with respect to Y. □

Theorem 38. *Let X be a continuum, the following sentences are equivalent.*

1. *X is pseudo-contractible.*
2. *For each compact metric space Y, X is pseudo-contractible with respect to Y.*
3. *For each compact metric space Z, Z is pseudo-contractible with respect to X.*
4. *$C(X, X)$ is continuumwise connected.*

Proof. (1) $\Rightarrow$ (2). Let Y be a compact metric space and let $f : X \to Y$ be a mapping. Since X is pseudo-contractible, then the identity mapping is pseudo-homotopic to a constant mapping. By Theorem 13, $f = f \circ id_X$ is pseudo-homotopic to a constant mapping, then X is pseudo-contractible with respect to Y.

(1) $\Rightarrow$ (3). Let Z be a compact metric space and let $g : Z \to X$ be a mapping. Since X is pseudo-contractible then the identity mapping is pseudo-homotopic to a constant mapping. By Theorem 13, $g = id_X \circ g$ is pseudo-homotopic to a constant, then Z is pseudo-contractible with respect to X.
The implications (2) $\Rightarrow$ (1) and (3) $\Rightarrow$ (1) are trivial.
Since X is a continuum, by Corollary 37 we have that (1) if and only if (4). ☐

Notice that if the space $C(X, X)$ is arcwise-connected then X is contractible and thus pseudo-contractible. The inverse implication does not hold. We consider the continuum X given by Kuperberg; it is pseudo-contractible but $C(X, X)$ is not arcwise-connected because if there was an arc joining the identity mapping with a constant mapping, then we could define a homotopy and thus X would be contractible, a contradiction.

Theorem 39. *Let X and Y be topological spaces. If X is pseudo-contractible with respect to Y and A is a retract of X then A is pseudo-contractible with respect to Y.*

Proof. Let $r : X \to A$ be a retraction from X to A and let $f : A \to Y$ be a mapping. Since X is pseudo-contractible with respect to Y, then $f \circ r : X \to Y$ is pseudo-homotopic to a constant mapping. On the other hand since $f = (f \circ r)|_A$, then by Remark 4, f is pseudo-homotopic to a constant mapping. ☐

As a consequence of the following sections we are going to obtain several obstructions to pseudo-contractibility and we get new pseudo-contractible and non pseudo-contractible spaces.

6. Pseudo-homotopy equivalent continua and pseudo-contractibility

Definition 40. *Let X and Y be topological spaces. We said that Y is semi-homotopy equivalent to X, written $Y \approx^{SE} X$, if there exist two mappings $g : Y \to X$ and $f : X \to Y$ such that $f \circ g \simeq id_Y$.*

Definition 41. *Let X and Y be topological spaces. We said that Y is semi-pseudo-homotopy equivalent to X, written $Y \approx^{SE}_{P} X$, if there exist two mappings $g : Y \to X$ and $f : X \to Y$ such that $f \circ g \simeq_C id_Y$.*

When the factor space C is equal to the interval $I = [0, 1]$, $Y \approx^{SE}_{P} X$ is equal to $Y \approx^{SE} X$.

Theorem 42. *Let Y and X be topological spaces. If Y is semi-pseudo-homotopy equivalent to X and X is pseudo-contractible, then so does Y.*

Proof. Since X is pseudo-contractible, we have that $id_X \simeq_C x_0$, where x_0 is a constant mapping.
Since Y is semi-pseudo-homotopy equivalent to X, there exist two mappings $g : Y \to X$ and $f : X \to Y$ such that $f \circ g \simeq_K id_Y$.
By Theorem 13, $f \circ id_X \simeq_C f \circ x_0$. Notice that $f = f \circ id_X$ and $y_0 = f \circ x_0 : X \to Y$ is a constant mapping.
Therefore by Theorem 13, we have $f \circ g \simeq_C y_0 \circ g$, where $y_0 \circ g : Y \to Y$ is a constant mapping.
Since $id_Y \simeq_K f \circ g$ and $f \circ g \simeq_C y_0 \circ g$, by Theorem 9, there exists a continuum D satisfying $id_Y \simeq_D y_0 \circ g$. Thus Y is pseudo-contractible. ☐

Theorem 43. *Let X, Z and Y be topological spaces. If X is pseudo-contractible with respect to Y and $Z \approx_P^{SE} X$, then Z is pseudo-contractible with respect to Y.*

Proof. Let $h : Z \to Y$ be a mapping. Since $Z \approx_P^{SE} X$ there are mappings $f : Z \to X$ and $g : X \to Z$ such that $g \circ f \simeq_K id_Z$. Then by Theorem 13, $h \circ g \circ f \simeq_K h \circ id_Z = h$.
On the other hand, since X is pseudo-contractible with respect to Y, $h \circ g \simeq_C y_0$ for some constant mapping, thus by Theorem 13, $h \circ g \circ f \simeq_C y_0 \circ f = y_0$. Therefore by Theorem 9, $h \simeq_D y_0$. ☐

Theorem 44. *Let X, Z and Y be topological spaces. If X is pseudo-contractible with respect to Y and $Z \approx_P^{SE} Y$, then X is pseudo-contractible with respect to Z.*

Proof. Let $h : X \to Z$ be a mapping. Since $Z \approx_P^{SE} Y$ there exist mappings $f : Z \to Y$ and $g : Y \to Z$ such that $g \circ f \simeq_K id_Z$. Then by Theorem 13, $g \circ f \circ h \simeq_K id_Z \circ h = h$.
On the other hand, since X is pseudo-contractible with respect to Y, $f \circ h \simeq_C y_0$, then by Theorem 13, $g \circ f \circ h \simeq_C g \circ y_0 = g(y_0)$. Therefore by Theorem 9, $h \simeq_D g(y_0)$. ☐

Definition 45. *Let X and Y be topological spaces. It said that X and Y are homotopy equivalent (or have the same homotopy type), written $X \approx^E Y$, if there exist two mappings $f : X \to Y$ and $g : Y \to X$ such that $f \circ g \simeq id_Y$ and $g \circ f \simeq id_X$.*

Definition 46. *Let X and Y be topological spaces. It said that X and Y are pseudo–homotopy equivalent (or have the same pseudo-homotopy type), written $X \approx_P^E Y$, if there exist two mappings $f : X \to Y$ and $g : Y \to X$ such that $f \circ g \simeq_K id_Y$ and $g \circ f \simeq_C id_X$.*

Corollary 47. *Let X and Y compact, metric spaces. If $X \approx_P^E Y$ and one of them is pseudo-contractible, then the other one is pseudo-contractible.*

The following theorem is easy to prove.

Theorem 48. *Let X be a topological space. X is pseudo-contractible if and only if X has the same pseudo-homotopy type as a point p.*

When the factor spaces C and K are equal to the interval $I = [0, 1]$, $Y \approx_P^E X$ is equal to $Y \approx^E X$.

7. Trivial Shape and Pseudo-contractibility

Definition 49. *A compact metric space K, is called an absolute neighborhood retract, written ANR, provided that whenever K is embedded in a metric space Y, the embedded copy K' of K is a retract of some neighborhood of K' in Y.*

Definition 50. *Let X be a continuum. We say that X has trivial shape provided that each mapping from X into an ANR space is homotopic to a constant mapping.*

It is well known the following result (see [4]).

Theorem 51. *Let X be a continuum, the following sentences are equivalents:*

1. *X has trivial shape.*
2. *X can be written as $X = \bigcap_{n\in\mathbb{N}} X_n$, where X_n is a contractible continuum for every $n \in \mathbb{N}$.*
3. *X can be written as an inverse limit of contractible continua.*
4. *For all $\varepsilon > 0$ there exists a contractible continuum Y_ε and an ε-map f_ε from X onto Y_ε.*

The following proposition appears in [18].

Proposition 52. *Let X be a compact metric space and let Y be an ANR space. If $f,\ g : X \to Y$ are pseudo-homotopic, then they are homotopic.*

Proposition 53. *Let X be a compact metric space and let Y be an ANR space. The space X is pseudo-contractible with respect to Y if and only if X contractible with respect to Y.*

Proof. Suppose that X is pseudo-contractible with respect to Y. Let $f : X \to Y$ be a mapping, then by Definition 34, f is pseudo-homotopic to a constant y_0. Since Y is an ANR space, by Proposition 52, f is homotopic to a constant y_0. Therefore X is contractible with respect to Y. The converse is trivial. $\square$

Note that if X is pseudo-contractible with respect to Y and Y is an ANR space (thus X contractible with respect to Y). Then Y is arcwise connected if and only if $C(X, Y)$ is arcwise connected.

Corollary 54. *Let X be an ANR space. Then X is pseudo-contractible if and only if X is contractible.*

Corollary 55. *Let X be a compact metric space. Then X has trivial shape if and only if X is pseudo-contractible with respect to each ANR space.*

Proof. Suppose that X is pseudo-contractible with respect to each ANR space. By Proposition 53, X is contractible with respect to each ANR space. Therefore X has trivial shape. $\square$

Theorem 56. *If X is a pseudo-contractible continuum then it has trivial shape.*

Proof. Since X is a pseudo-contractible continuum, then by Theorem 38, the space X is pseudo-contractible respect each compact metric space. In particular X is a pseudo-contractible with respect to each ANR space. Therefore by Corollary 55, X has trivial shape. $\square$

It is well known that S^1 does not have trivial shape. So, S^1 is not pseudo-contractible. Notice that if $X \approx^E S^1$, then by Corollary 47, X is not pseudo-contractible. The following continua are some examples of non pseudo-contractible continua because all of them have the same homotopy type that S^1.

1. The annulus $A = \{(x, y) : 1 \leq x^2 + y^2 \leq 2)\}$.
2. Solid Torus $S^1 \times D^2$.

3. Möbius strip.

In general, if X is a continuum such that it has no trivial shape and $Y \approx^E_P X$, then Y is not pseudo-contractible.

Notice that $S^1 \not\approx^E$ Warsaw circle, but they have the same shape.

Definition 57. *Let X and Y be topological spaces. A mapping $f : X \to Y$ is said to be pseudo-essential (essential) provided that f is not pseudo-homotopic (homotopic) to any constant mapping of X into Y. A mapping $f : X \to Y$ is said to be pseudo-inessential (inessential) provided that f is not pseudo-essential (essential).*

In this way, if $f : X \to Y$ is a pseudo-essential (essential) mapping, then X is not pseudo-contractible (contractible) with respect to Y. Note that if Y is an ANR space the notions of pseudo-essential mapping and essential mapping coincide.

Theorem 58. *Let X be a continuum. If X is a proper circle-like continuum, then X is not pseudo-contractible.*

Proof. Since X is a proper circle-like continuum, then by [10, Theorem 3.2] there exists an essential mapping from X onto S^1. Therefore X is not pseudo-contractible. ☐

Notice that the pseudo-circle is not pseudo-contractible because it is a proper circle-like continuum.

Theorem 51 and Theorem 56 imply the following result.

Corollary 59. *If X is a pseudo-contractible continuum then the following statements are true:*

1. *X has trivial shape.*
2. *X can be written as $X = \bigcap X_n$, where X_n's are contractible continua*
3. *X can be written as an inverse limit of contractible continua.*
4. *For all $\varepsilon > 0$, there exists a contractible continuum Y_ε and an ε-map f_ε from X onto Y_ε.*

The converse of this corollary is not true.

Example 60. *The sin* $\frac{1}{x}$ *curve satisfies the conditions of Corollary* 59, *but it is not pseudo-contractible (see [5]). Notice that if* $Y \approx_P^E sin\frac{1}{x}$ *then* Y *is not pseudo-contractible.*

8. Property *b*) and pseudo-contractibility

Definition 61. *Let* X *be a topological space. We say that* X *has the property b) provided that for each mapping* $f : X \to S^1$, *there exists a mapping* $g : X \to \mathbb{R}$ *such that* $f = \exp \circ g$, *where* $\exp : \mathbb{R} \to S^1$ *is defined by* $\exp(t) = (\cos(2\pi t), \sin(2\pi t))$ *and* $\mathbb{R}$ *denote the real line. The mapping* g *is called a lift of* f.

The following result is known.

Theorem 62. *[19, Theorem 6.2] Let* X *be a compact metric space. The space* X *is contractible with respect to* S^1 *if and only if* X *has the property b)*

As corollary of Proposition 53 and Theorem 62, we have the following.

Corollary 63. *Let* X *be a compact metric space. The following conditions are equivalents:*

1. X *is pseudo-contractible with respect to* S^1*;*
2. X *is contractible with respect to* S^1*;*
3. X *has the property b).*
4. $C(X, S^1)$ *is arcwise connected*

Theorem 64. *Let* X *be a compact metric space. If* X *is pseudo-contractible then* X *has property b).*

Proof. If X is pseudo-contractible, then by Theorem 38, X is pseudo-contractible with respect to S^1. Hence by Corollary 63, X has property b).

□

So, if X does not have Property b) then every space Y such that $Y \approx^E X$ is not pseudo-contractible.

Theorem 65. *[19, Theorem 7.3] Every connected space* X *having the property b) is unicoherent.*

Corollary 66. *Let X be a continuum. If X is pseudo-contractible then it is unicoherent.*

In this way, if X is not unicoherent then every space Y such that $Y \approx_P^E X$ is not pseudo-contractible.

Definition 67. *A continuum X is acyclic if $\check{H}^1(X, \mathbb{Z}) = 0$; i.e., the first Cĕch cohomology group with integer coefficients is trivial.*

By [6, Theorem 8.1] if a continuum X has property b) then it is acyclic. As a consequence we have the following result.

Corollary 68. *Let X be a continuum. If X is pseudo-contractible, then X is acyclic.*

So, if X is not acyclic, then every space Y such that $Y \approx_P^E X$, is not pseudo-contractible.

Finally, we are going to consider a continuum X, when X is a curve.

Theorem 69. *If X is a pseudo-contractible curve then it is hereditarily unicoherent.*

Proof. Let X be a curve. If X is pseudo-contractible, then by Theorem 56, X is a curve with trivial shape. Thus by [9, Theorem 2.1 (B)], X is tree like. Therefore by [2, Theorem 1], X is hereditarily unicoherent. □

For example we can see, using Theorem 69, that the following continua are not pseudo-contractible.

1. Menger sponge.
2. Sierpinski carpet.
3. Compactification of an arc with remainder a circle.

In general if X is a non hereditarily unicoherent curve. Then every space Y such that $Y \approx_P^E X$ is not pseudo-contractible.

Since Solenoids are hereditarily unicoherent, circle-like, and non acyclic curves, they are not pseudo-contractibles.

It is known that every hereditarily decomposable continuum is a curve, then we have the following result.

Corollary 70. *Let X be a hereditarily decomposable continuum. If X is pseudo-contractible then X is a λ-dendroid.*

The converse is not true, see Example 60.

Definition 71. *A metric space X is homogeneous provided that for each pair of points $x,\ y \in X$, there exists a homeomorphism $h : X \to X$ such that $h(x) = y$.*

Theorem 72. *Let X be a continuum. If X is a hereditarily decomposable pseudo-contractible continuum. Then X is not homogeneous.*

Proof. If X is homogeneous and hereditarily decomposable, by [14, Lemma 5.2], there exists an essential mapping from X onto S^1, a contradiction. ☐

In other words, every hereditarily decomposable homogeneous continuum is not pseudo-contractible or whether there is a homogeneous pseudo-contractible continuum it must be non hereditarily decomposable or equivalently the continuum must contains some indecomposable continuum.

Theorem 73. *Let X be a continuum. If X is a decomposable homogeneous curve then X is not pseudo-contractible.*

Proof. If X is a decomposable homogeneous curve, by [14, Lemma 5.1], there exists an essential mapping from X onto S^1. Therefore X is not pseudo-contractible. ☐

As a consequence we have that if X is a decomposable pseudo-contractible curve, then X is not homogeneous, or if there exist a homogeneous pseudo-contractible curve X, then X is indecomposable. Additionally, whether exists an homogeneous hereditarily equivalent pseudo-contractible continuum, it must be indecomposable.

Note that the circle of pseudo-arcs is not pseudo-contractible because it is a decomposable homogeneous curve.

Corollary 74. *Let X be a curve. The following propositions are true:*

1. *If X is pseudo-contractible with factor arcwise-connected space, then X is a uniformly arcwise-connected dendroid. Moreover. The curve X is contractible and it is a uniformly arcwise connected dendroid.*
2. *If X is pseudo-contractible and arcwise-connected then X is a dendroid.*
3. *The space X is locally connected and pseudo-contractible if and only if X is a dendrite.*

The converses of Corollary 74.1. and Corollary 74.2. are not true. We consider the continuum given in [18, Corollary 5]. The continuum X is a uniformly arcwise-connected dendroid but it is not pseudo-contractible. This continuum is pictured here.

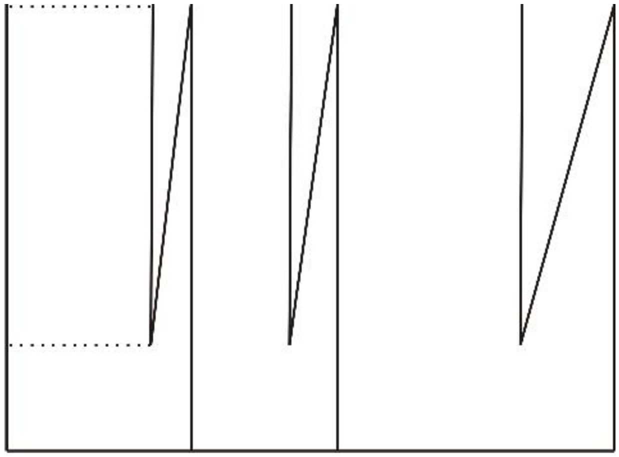

Figure 2: Non pseudo-contractible dendoid.

Since every pseudo-contractible and arcwise-connected curve is a dendroid, is natural to ask the following question.

Question 75. *[3, Question 4.10] Is every pseudo-contractible dendroid also contractible?*

In general:

Question 76. *[12, Problem 118] Does there exist a curve which is pseudo-contractible but not contractible?*

Question 77. *Does there exist an arcwise connected continuum which is pseudo-contractible but not contractible? Or equivalently, every pseudo-contractible arcwise connected continuum is contractible?*

Sobolewski in [18] proves that the only non degenerate chainable pseudo-contractible continuum is the arc. In particular, the pseudo-arc and the Knaster-type indecomposable continua are not pseudo-contractible.

Question 78. *[17, Question 19, pp 309] Does there exist a nondegenerate (hereditarily) indecomposable continuum which is pseudo-contractible?*

Acknowledgments

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Capítulo 4

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Dear Professor Felix Capulin,

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R^i-SETS, HYPERSPACES AND PSEUDO-CONTRACTIBILITY ON CONTINUA

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Abstract. In this paper we discuss the notion of pseudo-contractibility on hyperspaces of continua. Also we prove that if a continuum X contains an R^i-set then it is not pseudo-contractible. As a consequence we have that the existence of an R^i-set in a continuum X implies non(pseudo)-contractibility of some hyperspaces.

1. Introduction

R. H. Bing introduced the notion of pseudo-contractibility. W. Kuperberg gave the first example which proves that the notions of pseudo-contractibility and contractibility are different. This example was never published by himself but is known among continuum theorists. He asked whether or not the $\sin \frac{1}{x}$ curve is pseudo-contractible. H. Katsuura proves in [11] that the $\sin\frac{1}{x}$ curve is not pseudo-contractible with factor space itself. In the same paper H. Katsuura shows that if Y is a nondegenerate indecomposable continuum such that each one of its composants is arcwise connected and X is a continuum that has a proper nondegenerate arc component, then X is not pseudo-contractible with factor Y.

W. Dębski shows in [7] that the $\sin \frac{1}{x}$ curve is not pseudo-contractible. On the other hand, M. Sobolewsky proves that the only chainable continuum that is pseudo-contractible is the arc, (see [21]), in particular the pseudo-arc is another example of a non pseudo-contractible continuum. In [3] there is a general study about pseudo-homotopies and pseudo-contractibility. The interested reader is referred to [2], [3], [7], [9], [11] and [21]. The present paper is divided in five sections. After the introduction and preliminaries,

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Key words and phrases. Continuum, hyperspace, induced mapping, quotient space.

in section 3 we present results about pseudo-contractibilty and hyperspaces, mainly we show that the concepts of pseudocontractibility and contractibility are equivalent in some hyperspaces. In section 4 we prove that if a continuum X contains an R^i-set then it is not pseudo-contractible. As a consequence we have that the existence of an R^i-set in a continuum X implies non(pseudo)-contractibility of some hyperspaces. In section 5 we present some questions about pseudo-contractibility and hyperspaces.

2. PRELIMINARIES

A *continuum* means a nonempty compact connected metric space. A *map* is a continuous function. If there exists a homeomorphism $f : X \to Y$, we say that X is homeomorphic to Y and we write $X \approx Y$. A continuum X is said to be *unicoherent* provided that for each pair of subcontinua H and K of X such that $X = H \cup K$, $H \cap K$ is connected, and it is *hereditarily unicoherent* if each subcontinuum of X is unicoherent. An *arc* is understood as a homeomorphic image of a closed unit interval $I = [0, 1]$. A space Z is said to be *arcwise-connected* provided each pair of points of a space Z can be joined by an arc lying in Z. A *curve* is a one-dimensional continuum.

A continuum X is *arc-like (circle-like)* provided that for each $\varepsilon > 0$, there exists an ε-map $f : X \to [0, 1]$ ($f : X \to S^1$, where S^1 is the unit circle). A *proper circle-like continuum* is a circle-like continuum which is not an arc-like continuum.

Let X be a continuum. The hyperspace of all nonempty closed subsets of X is denoted by 2^X, the hyperspace of all subcontinua of X is denoted by $C(X)$, if $n \in \mathbb{N}$, the hyperspace of all nonempty closed subsets of X with at most n components is denoted by $C_n(X)$, the hyperspace of all nonempty subsets of X with at most n points is denoted by $F_n(X)$, in particular $F_1(X)$ is called the hyperspace of singletons of X and it is homeomorphic to X, $F_\infty(X)$ denotes the hyperspace of all finite subsets of X and $C_\infty(X)$ denotes the hyperspace of all closed subsets of X with a finite number of components. Note that $C_1(X) = C(X)$, $F_1(X) \subset C(X) \subset C_n(X) \subset C_\infty(X) \subset 2^X$, $F_1(X) \subset F_n(X) \subset F_\infty(X) \subset C_\infty(X) \subset 2^X$ and $F_n(X) \subset C_n(X)$. These hyperspaces are topologized with the *Hausdorff metric* defined by: $\mathcal{H}_d(A, B) = \inf\{\varepsilon > 0 | A \subset N_\varepsilon(B) \text{ and } B \subset N_\varepsilon(A)\}$, where $N_\varepsilon(Y) = \{x \in X |$ there exists $a \in Y$ such that $d(y, a) < \varepsilon\}$ for $\varepsilon > 0$ and $Y \in 2^X$. It is well known that $\mathcal{H}_d(A, B) < \epsilon$ if and only if $A \subset N_\varepsilon(B)$ and $B \subset N_\varepsilon(A)$.

The following are the usual definitions related with contractibility (see for example [22, p. 225], [13, pp. 370, 374], [10, pp. 155, 156], [5, p. 748]).

DEFINITION 2.1. *Let X and Y be topological spaces and let f, $g : X \to Y$ be maps. We say that f is homotopic to g (or f and g are homotopic), written $f \simeq g$, if there exists a map (called homotopy) $H : X \times I \to Y$ satisfying $H(x, 0) = f(x)$ y $H(x, 1) = g(x)$ for each $x \in X$.*

DEFINITION 2.2. *A topological space X is said to be:*
a) contractible if its identity map is homotopic to a constant map in X,
b) contractible with respect to a topological space Y if each map $f : X \to Y$ is homotopic to a constant map.
A subspace Z of X is said to be contractible in X if the inclusion map from Z into X is homotopic to a constant map in X.

It is not difficult to prove that X is contractible if and only if X is contractible with respect to Y, for every space Y.

Following the classical definitions of contractibility, concepts related with pseudo-contractibility are defined as follows:

DEFINITION 2.3. *Let X and Y be topological spaces, let C be a continuum and let $f,\ g : X \to Y$ be maps. We say that f is pseudo-homotopic to g with factor space C(or f and g are pseudo-homotopic with factor space C) if there are two points $a,\ b \in C$ and a map $H : X \times C \to Y$ such that $H(x, a) = f(x)$ and $H(x, b) = g(x)$ for each $x \in X$. We write $f \simeq_C g$ to say that f is pseudo-homotopic to g with factor space C. The map H is called a pseudo-homotopy between f and g with factor space C. Clearly if C is homeomorphic to the unit interval $I = [0, 1]$, $f \simeq_C g$ is equivalent to $f \simeq g$.*

DEFINITION 2.4. *A topological space X is said to be:*
a) pseudo-contractible if its identity map is pseudo-homotopic to a constant map in X,
b) pseudo-contractible with respect to a topological space Y if each map $f : X \to Y$ is pseudo-homotopic to a constant map.
A subspace Z of X is said to be pseudo-contractible in X if the inclusion map from Z into X is pseudo-homotopic to a constant map in X.

It is clear that in each case contractibility implies pseudo-contractibility.

REMARK 2.5. Let $W \subset Z \subset X$. If Z is pseudo-contractible in X, then W is pseudo-contractible in X. If Z is pseudo-contractible in X and $X \subset X'$ then Z is pseudo-contractible in X'. If X is pseudo-contractible, then every subspace Z of X is pseudo-contractible in X.

3. HYPERSPACES AND PSEUDO-CONTRACTIBILITY.

In [20] the author studies the concepts of contractibility and weak contractibility for compact connected Hausdorff spaces (nonmetric continua) and their hyperspaces. He used the same ideas used in metric continua to define contractibility and weak contractibility in nonmetric continua (the factors spaces in those concepts are nonmetric arcs and nonmetric continua) as well as we have defined here contractibility and pseudo-contractibility with metric continua. These concepts are different, however both concepts are equivalent when we use metric continua.

In [20, Theorems 3 and 4 and Corollary] D. G. Paulowich proves that if X is a compact space such that $F_1(X)$ is a retract of $C(X)$, Y is a compact subspace of X and Y is weakly contractible in X, then Y is contractible in X. Also he proves that if X be a non metric continuum, the following three statements are equivalent:

1. $F_1(X)$ is contractible in 2^X.
2. 2^X is contractible.
3. $C(X)$ is contractible.

As a corollary he proves that if X is a weak contractible non metric continuum, then 2^X and $C(X)$ are contractible.

In particular we get as a consequence of [20, Theorems 3] that if X is a compact space such that $F_1(X)$ is a retract of $C(X)$ and X is weakly contractible, then X is contractible.

Notice that Theorem 3, Theorem 4 and Corollary in [20, p. 44] are true if we put contractibility and pseudo-contractibility, in the sense of this work, instead of contractibility and weak contractibility in the sense of [20].

As a corollary, we have that, if X is contractible in metric continua, then $C(X)$ and 2^X are contractible (see also [19, Corollary 16.8]).

The main problem in this section is to prove that the notions of pseudo-contractibility and contractibility coincide in the hyperspaces 2^X, $C(X)$, $C_\infty(X)$ and $C_n(X)$ for any $n \in \mathbb{N}$.

THEOREM **3.1.** *Let X be a continuum. If $F_1(X)$ is pseudo-contractible in 2^X, then 2^X is pseudo-contractible.*

PROOF. Assume that there exist a continuum C, points $a,\ b \in C$ and a map $H : F_1(X) \times C \to 2^X$, such that $H(\{k\}, a) = \{k\}$ and $H(\{k\}, b) = A_0$ for some $A_0 \in 2^X$ and each $\{k\} \in F_1(X)$.
Consider the function $G : 2^X \times C \to 2^X$ defined by $G(K, c) = \bigcup\{H(\{k\}, c) : k \in K\}$. By [19, 1.48], G is well defined and $G(K, a) = \bigcup\{H(\{k\}, a) : k \in K\} = \bigcup\{\{k\} : k \in K\} = K$, and $G(K, b) = \bigcup\{H(\{k\}, b) : k \in K\} = A_0$ for each $K \in 2^X$.
We need to show that G is continuous.
Let $\varepsilon > 0$. Since H is uniformly continuous, there exists $\delta > 0$ such that if $\mathcal{H}_d(\{p\}, \{q\}) < \delta$ and $d(s, t) < \delta$ then $\mathcal{H}_d(H(\{p\}, t), H(\{q\}, s)) < \varepsilon$. Let $A,\ B \in 2^X$ and let $s,\ t \in C$ such that $\mathcal{H}_d(A, B) < \delta$ and $d(s, t) < \delta$. We will show that $\mathcal{H}_d(G(A, t), G(B, s)) < \varepsilon$, by proving that $G(A, t) \subset N_\varepsilon(G(B, s))$ and $G(B, s) \subset N_\varepsilon(G(A, t))$.
Let $p' \in G(A, t)$, then there exists a point $p \in A$ such that $p' \in H(\{p\}, t)$. Since $A \subset N_\delta(B)$ there exists $q \in B$ such that $d(p, q) < \delta$. So, $\mathcal{H}_d(\{p\}, \{q\}) < \delta$. On the other hand since $d(t, s) < \delta$ we have that $\mathcal{H}_d(H(\{p\}, t), H(\{q\}, s)) < \varepsilon$. Therefore, $p' \in H(\{p\}, t) \subset N_\varepsilon(H(\{q\}, s)) \subset N_\varepsilon(G(B, s))$. Thus $p' \in$

$N_\varepsilon(G(B,s))$. Hence $G(A,t) \subset N_\varepsilon(G(B,s))$. Analogously $G(B,s) \subset N_\varepsilon(G(A,t))$. Thus G is continuous and therefore, 2^X is pseudo-contractible. □

THEOREM 3.2. *Let X be a non metric continuum. If 2^X is weak contractible then it is contractible (in the sense of [20]).*

PROOF. If 2^X is weak contractible, then by Theorem [20, Corollary, p. 44], 2^{2^X} is contractible and since $F_1(2^X)$ is a retract of 2^{2^X} ([12, Lemma 1.1]) we have by [3, Theorem 28, p. 9] that $F_1(2^X)$ is contractible. Notice that $F_1(2^X) \approx 2^X$, thus 2^X is contractible. □

COROLLARY 3.3. *Let X be a continuum. If 2^X is pseudo-contractible then it is contractible.*

Notice that using the fact that $F_1(2^Z)$ is a retract of $C(2^Z)$, we prove also that $F_1(Z)$ is contractible in 2^Z if $F_1(Z)$ is pseudo-contractible in 2^Z, taking X as 2^Z and Y as $F_1(Z)$ in [20, Theorem 3].

COROLLARY 3.4. *Let X be a continuum. If $F_1(X)$ is pseudo-contractible in 2^X then 2^X is contractible.*

COROLLARY 3.5. *Let X be a continuum. If $F_1(X)$ is pseudo-contractible in $C_\infty(X)$ then $C_\infty(X)$ is contractible (therefore, pseudo-contractible).*

PROOF. Since $F_1(X)$ is pseudo-contractible in $C_\infty(X)$, $F_1(X)$ is pseudo-contractible in 2^X. By Corollary 3.4, 2^X is contractible. Thus by [16, Theorem 8.7], $C_\infty(X)$ is contractible (therefore, pseudo-contractible). □

COROLLARY 3.6. *Let X be a continuum and let $n \in \mathbb{N}$. If $F_1(X)$ is pseudo-contractible in $C_n(X)$, then $F_1(X)$ is contractible in $C_n(X)$.*

PROOF. Since $F_1(X)$ is pseudo-contractible in $C_n(X)$, $F_1(X)$ is pseudo-contractible in 2^X. By Corollary 3.4, 2^X is contractible. So, by [20, Theorem 4], $C(X)$ is contractible and by [8, Exercise 9.7], $F_1(X)$ is contractible in $C_n(X)$. □

COROLLARY 3.7. *Let X be a continuum. If $C(X)$ is pseudo-contractible then it is contractible.*

PROOF. If $C(X)$ is pseudo-contractible then $F_1(X)$ is pseudo-contractible in 2^X. By Corollary 3.4, 2^X is contractible and by [20, Theorem 4], $C(X)$ is contractible. □

COROLLARY 3.8. *Let X be a continuum and let $n \in \mathbb{N}$. If $C_n(X)$ is pseudo-contractible then $C_n(X)$ is contractible.*

PROOF. If $C_n(X)$ is pseudo-contractible then $F_1(X)$ is pseudo-contractible in $C_n(X)$ and thus $F_1(X)$ is pseudo-contractible in 2^X and by Corollary 3.4, 2^X is contractible, by [20, Theorem 4], $C(X)$ is contractible and by [8, Exercise 9.7], $C_n(X)$ is contractible. □

The following corollary is a consequence of [20, Theorem 4], [8, Exercise 9.7], Theorem 3.1, and Corollaries 3.3, 3.4, 3.5, 3.6, 3.7 and 3.8.

COROLLARY 3.9. *Let X be a continuum, the following sentences are equivalent:*

1. $F_1(X)$ *is contractible in* $C(X)$.
2. $F_1(X)$ *is contractible in* 2^X.
3. 2^X *is contractible.*
4. $C(X)$ *is contractible.*
5. $F_1(X)$ *is contractible in* $C_n(X)$ *for some* $n \in \mathbb{N}$.
6. $C_n(X)$ *is contractible for each* $n \in \mathbb{N}$.
7. $C_n(X)$ *is contractible for some* $n \in \mathbb{N}$.
8. $F_1(X)$ *is contractible in* $C_\infty(X)$.
9. $C_\infty(X)$ *is contractible.*
10. $F_1(X)$ *is pseudo-contractible in* 2^X.
11. $F_1(X)$ *is pseudo-contractible in* $C(X)$.
12. $C(X)$ *is pseudo-contractible.*
13. 2^X *is pseudo-contractible.*
14. $F_1(X)$ *is pseudo-contractible in* $C_n(X)$ *for some* $n \in \mathbb{N}$
15. $C_n(X)$ *is pseudo-contractible for each* $n \in \mathbb{N}$.
16. $C_n(X)$ *is pseudo-contractible for some* $n \in \mathbb{N}$.
17. $F_1(X)$ *is pseudo-contractible in* $C_\infty(X)$.
18. $C_\infty(X)$ *is pseudo-contractible.*

COROLLARY 3.10. *Let X be a continuum. If X is pseudo-contractible then we have that propositions (1)-(18) of Corollary 3.9 hold.*

The converse of Corollary 3.10, is not true, we consider the unicoherent continuum $X = X_0 \cup \{x : |x| = 1\} \subset \mathbb{C}$, where $X_0 = \{\frac{t+2}{t+1}e^{it} : t \in [0, \infty)\}$ is the spiral approaching the unit circle S^1 (the symbol $\mathbb{C}$ denotes the set of the complex numbers). We know that $C(X)$ is contractible, because $C(X)$ is homeomorphic to the $cone(X)$. However X is not pseudo-contractible, because X is a curve, i.e., if X were pseudo-contractible, thus by [3, Theorem 69, p. 19] it would be hereditarily unicoherent, a contradiction. On the other hand S^1 is a non unicoherent continuum such that $C(S^1)$ is contractible, but by [3, Theorem 69, p. 19], S^1 is not pseudo-contractible.

Now we discuss the pseudo-contractibility in the hyperspaces $F_n(X)$ and $F_\infty(X)$.

THEOREM 3.11. *Let X be a continuum. $F_1(X)$ is pseudo-contractible in $F_\infty(X)$ if and only if $F_\infty(X)$ is pseudo-contractible.*

PROOF. Assume that there exist a continuum C, points $a,\ b \in C$ and a map $H : F_1(X) \times C \to F_\infty(X)$, such that $H(\{k\}, a) = \{k\}$ and $H(\{k\}, b) = A_0$

for some $A_0 \in F_\infty(X)$ and each $\{k\} \in F_1(X)$.
Consider the function $G : F_\infty(X) \times C \to F_\infty(X)$ defined by $G(K, c) = \bigcup\{H(\{k\}, c) : k \in K\}$. Notice that $G(K, c) \in F_\infty(X)$ because K is a finite subset of X. Since H is a map, thus by [19, 1.48], G is well defined and we have that:
$G(K, a) = \bigcup\{H(\{k\}, a) : k \in K\} = \bigcup\{\{k\} : k \in K\} = K$.
$G(K, b) = \bigcup\{H(\{k\}, b) : k \in K\} = A_0$ for each $K \in F_\infty(X)$.
The continuity of G, can be proved as in Theorem 3.1.
The other implication is trivial. □

COROLLARY 3.12. *Let X be a continuum. If X is a pseudo-contractible continuum then $F_\infty(X)$ is pseudo-contractible.*

The inverse of this result is not true. By [8, Exercise 9.8], $F_\infty(S^1)$ is contractible but S^1 is not pseudo-contractible.

COROLLARY 3.13. *Let X be a continuum. If $F_\infty(X)$ is pseudo-contractible then we have that propositions (1)-(18) of Corollary 3.9 hold.*

PROOF. Since $F_\infty(X)$ is pseudo-contractible, $F_1(X)$ is pseudo-contractible in $F_\infty(X)$ and therefore, $F_1(X)$ is pseudo-contractible in 2^X. □

COROLLARY 3.14. *Let X be a continuum. If $F_1(X)$ is pseudo-contractible in $F_n(X)$ for some $n \in \mathbb{N}$, then we have that propositions (1)-(18) of Corollary 3.9 hold.*

THEOREM 3.15. *Let X be a continuum, if X is pseudo-contractible, then $F_n(X)$ is pseudo-contractible for any $n \in \mathbb{N}$.*

PROOF. Since X is pseudo contractible, there exist a continuum C, points $a, b \in C$ and a map $H : X \times C \to X$ such that $H(x, a) = x$ and $H(x, b) = x_0$ for each $x \in X$.
Let $n \in \mathbb{N}$, the function $G : F_n(X) \times C \to F_n(X)$ defined by $G(\{x_1, \ldots, x_n\}, c) = \{H(x_1, c), \ldots, H(x_n, c)\}$ is a pseudo-homotopy between the identity in $F_n(X)$ and the constant map $\{x_0\}$, i.e., $G(\{x_1, \ldots, x_n\}, a) = \{H(x_1, a), \ldots, H(x_n, a)\} = \{x_1, \ldots, x_n\}$ and $G(\{x_1, \ldots, x_n\}, b) = \{H(x_1, b), \ldots, H(x_n, b)\} = \{x_0, \ldots, x_0\}$. □

COROLLARY 3.16. *Let X be a continuum. If $F_n(X)$ is pseudo-contractible for some $n \in \mathbb{N}$ then we have that propositions (1)-(18) of Corollary 3.9 hold and also $F_\infty(X)$ is pseudo-contractible.*

PROOF. If $F_n(X)$ is pseudo-contractible, then $F_1(X)$ is pseudo-contractible in $F_\infty(X)$ and by Theorem 3.11, $F_\infty(X)$ is pseudo-contractible. On the other hand $F_1(X)$ pseudo-contractible in $F_\infty(X)$ implies that $F_1(X)$ is pseudo-contractible in 2^X. □

The converse of Corollary 3.16 is not true. We give two examples; one of them is a non unicoherent continuum with $F_n(X)$ non pseudo-contractible for every $n > 1$ and the other one is an unicoherent continuum with $F_2(X)$ non pseudo-contractible. To see the first example, $X = S^1$ is a non unicoherent continuum, and it is known by [8, Exercise 9.8] that $F_\infty(S^1)$ is contractible, and (1)-(18) hold because $C(S^1)$ is contractible. On the other hand, we know that if $n > 1$, $F_{2n+1}(S^1)$ is homotopically equivalent to S^{2n+1} (see [6, Theorem 4.1]) and $F_{2n}(S^1)$ is homotopy equivalent to S^{2n-1} (see [6, Theorem 4.2]). Since S^n is ANR for all $n \in \mathbb{N}$ and S^n is not contractible, then by [3, Corollary 54, p. 16], S^n is not pseudo-contractible. Therefore, by [3, Corollary 47, p. 14], $F_n(S^1)$ is not pseudo-contractible for all $n \in \mathbb{N}$.
For the second example we consider $X = S_1 \cup Y \cup S_2 \subset \mathbb{C}$, where
$Y = \{(\dfrac{t}{1+|t|}+2)e^{it} : t \in \mathbf{R}\}$, $S_1 = \{e^{it} : t \in \mathbb{R}\}$ and $S_2 = \{3e^{it} : t \in \mathbf{R}\}$.
X is the union of two circles and a spiral which surrounds them asymptotically. $C(X)$ is contractible, because X is a Kelley's continuum (see [8, Theorem 9.4, p. 129]). By [4], $F_2(X)$ is not unicoherent and by [3, Corollary 66, p. 18], it is not pseudo-contractible.

4. R^i-SETS AND PSEUDO-CONTRACTIBILITY

By [1, Corollary 3.3] if a continuum X has an R^i-set $i = 1, 2, 3$, then X is not contractible. The main result of this section is to prove that if a continuum X has an R^i-set, then X is not pseudo-contractible.

DEFINITION 4.1. *Let* $\{A_n\}_{n\in\mathbb{N}}$ *be a sequence of subsets of a space* X.
$\liminf A_n = \{x \in X :$ *for each open* $U \subset X$ *such that* $x \in U$, $U \cap A_n \neq \emptyset$ *for all but finitely many* $n\}$.
$\limsup A_n = \{x \in X :$ *for each open* $U \subset X$*such that* $x \in U$, $U \cap A_n \neq \emptyset$ *for infinitely many* $n\}$.
Let $A \in 2^X$, *we write* $\lim A_n = A$ *to mean* $\liminf A_n = A = \limsup A_n$.

DEFINITION 4.2. *A nonempty closed proper subset* K *of a continuum* X *is called;*

- R^1*-set if there exist an open set* U *containing* K *and two sequences* $\{C_n^i\}_{n\in\mathbb{N}}$, $i = 1,\ 2$ *of components of* U *such that* $K = \limsup C_n^1 \cap \limsup C_n^2$.
- R^2*-set if there exist an open set* U *containing* K *and two sequences* $\{C_n^i\}_{n\in\mathbb{N}}$, $i = 1,\ 2$, *of components of* U *such that* $K = \lim C_n^1 \cap \lim C_n^2$.
- R^3*-set if there exist an open set* U *and a sequence* $\{C_n\}_{n\in\mathbb{N}}$ *of components of* U *such that* $K = \liminf C_n$.

The following theorem is due to W. J. Charatonik and discussed in México where we reviewed some details to the final version.

THEOREM 4.3. *Let X be a continuum. If X contains an R^3-set, then X is not pseudo-contractible.*

PROOF. Let A be an R^3-set of X. By definition, there exist an open set U and a sequence $\{C_n\}_{n\in\mathbb{N}}$ of components of U such that $A = \liminf C_n$.
Since $A \subset U$ there exists an $\varepsilon > 0$ such that $d(A, X \setminus U) > \varepsilon$.
Suppose that $H : X \times Y \to X$ is a pseudo-contraction of X, i.e. there exist a continuum Y and points $a, b \in Y$ such that $H(x, a) = x$ and $H(x, b) = x_0$ for all $x \in X$ and $x_0 \in X$.
Since H is uniformly continuous, there exists $\delta > 0$ such that if $\operatorname{diam}(K) < \delta$ then $\operatorname{diam}(H(K \times \{t\})) < \varepsilon$, for each $t \in Y$.
Now consider the set $P = \{c, c_1, \dots, c_n, \dots\} \subset U$, where $c \in A$, $c_i \in C_i$ for each $i > 0$ and $\lim c_i = c$. Without loss of generality we can assume that $\operatorname{diam}(P) < \delta$. Let $V = \{t \in Y : H(P \times \{t\}) \subset U\}$. The set V satisfies the following conditions:

1. $V \neq \emptyset$. because $a \in V$.
2. $V \neq Y$.
3. V is an open set of Y.

The set $V \neq \emptyset$ because $a \in V$. $V \neq Y$ because if $V = Y$ then $b \in V$ and $H(P \times \{b\}) = x_0 \in U$. If C is the component of U containing x_0, we can consider C_j a component of U such that $C_j \neq C$. Since $c_j \in C_j$, $H(\{c_j\} \times Y) \subset U$ and $H(\{c_j\} \times Y)$ is a connected set containing x_0 and c_j, a contradiction. Therefore, $V \neq Y$. Finally, by the tube Lemma, (see [17, Lemma 5.8, p.169]) V is an open set of Y.
Let V_0 be the component of V containing a. Since $H(\{c_i\} \times V_0)$ is a connected set containing c_i, then $H(\{c_i\} \times V_0) \subset C_i$ for all $i = 1, 2, \dots$ In other words, $H((c_i, t) \in C_i$ for all $i = 1, 2, \dots$ and all $t \in V_0$.
Since $\lim H(c_i, t) = H(c, t)$, then $H(c, t) \in \liminf C_i = A$ for all $t \in V_0$.
On the other hand, if $t_0 \in Bd(V_0)$ there exists a sequence $\{t_n\}_{n\in\mathbb{N}} \subset V_0$ such that $\lim t_n = t_0$. So, $\lim H(c, t_n) = H(c, t_0) \in A$, because A is a closed set.
Finally, since V_0 is a component of the open set V, by [18, 5.7 Boundary Bumping Theorem III, p. 75], $\overline{V_0} \cap (Y \setminus V) \neq \emptyset$. Let $t' \in \overline{V_0} \cap (Y \setminus V)$, then $t' \in Bd(V_0) \setminus V$, i.e., $H(P \times \{t'\}) \not\subset U$; in this way, there exists $d \in P$ such that $H(d, t') \notin U$, but $H(c, t') \in A$. Therefore, $d(H(c, t'), H(d, t')) > \varepsilon$, a contradiction, because $c,\ d \in P$ and $\operatorname{diam}(H(P \times \{t'\})) < \varepsilon$. ☐

It is known by [1, Theorem 2.3] that each R^2-set is an R^3-set and by [1, Theorem 2.5], each R^1-set contains an R^3-set.

COROLLARY 4.4. *Let X be a continuum. If X is pseudo-contractible, then X contains no R^i-set for $i = 1, 2, 3$.*

THEOREM 4.5. *Let X be a continuum. If X has an R^3-set, then*

1. 2^X *is not (pseudo-)contractible.*

2. *$C(X)$ is not (pseudo-)contractible.*
3. *$C_n(X)$ is not (pseudo-)contractible for each $n \in \mathbb{N}$.*
4. *$C_\infty(X)$ is not (pseudo-)contractible.*
5. *$F_n(X)$ is not pseudo-contractible for each $n \in \mathbb{N}$.*
6. *$F_\infty(X)$ is not pseudo-contractible.*

Proof. If X has an R^i-set for $i = 1, 2, 3$, then by [1, Theorem 3.6] and [1, Theorem 3.7], 2^X and $C(X)$ contain an R^i-set for $i = 1, 2, 3$. Hence by Theorem 4.3 they are not (pseudo-)contractible.
Let $\mathbb{H}(X) \in \{F_\infty(X),\ C_\infty(X),\ F_n(X),\ C_n(X)\}$. If X has an R^3 set and we assume that $\mathbb{H}(X)$ is pseudo-contractible then by Corollaries 3.9, 3.13 and 3.16 , 2^X is pseudo-contractible, a contradiction. □

Finally we see some continua X such that their $F_2(X)$ is not pseudo-contractible. First of all we recall the following definition.

Definition 4.6. *Let X and Y be topological spaces. A map $f : X \to Y$ is said to be essential provided that f is not homotopic to any constant map of X into Y. A map $f : X \to Y$ is said to be inessential provided that f is not essential.*

It is well known by [15, Theorem 3.4] that if X is a proper circle-like continuum, then $F_2(X)$ does not have trivial shape. Therefore, if X is a proper circle-like continuum, then by [3, Theorem 56, p.16], $F_2(X)$ is not pseudo-contractible.

On the other hand, it is known by [15, Theorem 3.3] that if there exists an essential map from a continuum X onto S^1, then there exists an essential map from $F_2(X)$ onto S^1, in other words, $F_2(X)$ does not have trivial shape and therefore, by [3, Theorem 56, p.16], $F_2(X)$ is not pseudo-contractible. In particular $F_2(X)$ is not pseudo-contractible if X is a decomposable homogeneous curve or hereditarily decomposable homogeneous continuum, since by [14, Lemma 5.1] and [14, Lemma 5.2] there exists an essential map from X onto S^1.

5. Questions

In Section 3, we have showed that for any continuum X the concepts of pseudo-contactibility and contractibility are the same in the hyperspaces 2^X, $C(X)$, $C_n(X)$ and $C_\infty(X)$.
Let $\mathbb{H}(X) \in \{F_\infty(X),\ F_n(X),\ C_\infty(X),\ C_n(X),\ 2^X\}$, $n \in \mathbb{N}$. The following question appears in [5, Question 3.11, p. 755].

(1) Let X be a continuum. What are necesary and /or sufficient conditions in terms of X in order that $\mathbb{H}(X)$ is or is not contractible?
Partial answer is given in Corollary 3.10.

Now let's consider $\mathbb{H}(X) \in \{F_\infty(X),\ F_n(X)\}$, $n \in \mathbb{N}$.

(2) Let X be a continuum. What are necesary and /or sufficient conditions in terms of X in order that $\mathbb{H}(X)$ is or is not pseudo-contractible?

Partial answers are given in Corollary 3.12 and Theorem 3.15.

(3) $\mathbb{H}(X)$ pseudo-contractible implies $\mathbb{H}(X)$ contractible for any continuum X?

(4) Let X be a continuum. If $F_n(X)$ is pseudo-contractible for some $n \in \mathbb{N}$, then is X pseudo-contractible?

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Capítulo 5

Conclusiones

Durante la de investigación se abordaron los 3 puntos planteados en el problema de investigación obteniendo principalmente lo siguiente:

1. En el artículo “General properties on Pseudo-contractibility” se dan condiciones para que un espacio sea o no pseudocontráctil.

2. En el artículo “Selectibility is not preserve under open light mappings between fans” se da un ejemplo de un espacio selectible tal que la imagen abierta y ligera no es selectible.

3. Por último, se llegó a la conclusión de que la existencia de R^3-conjuntos, impide la pseudocontractibilidad del espacio. además de ello se demostró que si el espacio X es pseudocontraćtil entonces los hiperespacios 2^X, $C_n(X)$, $C_\infty(X)$, $F_\infty(X)$ y $C(X)$ son contraćtiles, quedando pendiente el análisis de algunos otros hiperespacios como $F_n(X)$. Los resultados anterirmente mencionados aparecen en el articulo “R^i- sets, hyperspaces and pseudo-contractibility on continua”.

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